UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematics for Sustanaible Scenarios

2627-2-F7603Q033-F7603Q03302

Learning objectives

The course aims to provide students with the mathematical dynamical tools necessary to analyze and understand the temporal evolution of complex systems in ecological, economic, and socio-environmental contexts. Starting from the fundamentals of differential equations and difference equations, students will acquire the ability to mathematically formalize the dynamics of populations, natural resources, and economic variables, understanding the crucial role of nonlinearity in generating qualitatively and quantitatively different behaviors: stable or unstable equilibria, periodic oscillations, bifurcations, and deterministic chaos.

In parallel, the course introduces students to the historical evolution of modeling thought, from Malthus (1798) to Verhulst (1838), Gompertz (1825), Lotka-Volterra (1920-1926), up to the most recent integrated models such as World3 (1972), Brander-Taylor (1998), Pezzey-Anderies (2001), and HANDY (2014). Students will become aware of the heuristic value of mathematical models as "thought experiments" to explore sustainability or collapse scenarios, as well as the intrinsic limitations of each modeling approach.

A qualifying element of the course is the integration of biodiversity as a cross-cutting theme: students will understand the role of biological diversity as the foundation of ecosystem services and as a factor of resilience, as well as the modeling tools (species-area relationship, metapopulations, spatial models) to quantify the effects of habitat fragmentation and environmental degradation.

By the end of the course, students will be able to:

- Mathematically formalize simple dynamical problems in ecological and economic contexts, translating qualitative relationships between variables (population, resources, wealth, pollution) into differential or difference equations.

- Analyze the dynamics of single- or multi-variable models, determining equilibrium points, studying their stability through linear analysis (eigenvalues for systems of ODEs, derivatives for one-dimensional maps), and interpreting the results in terms of system sustainability or vulnerability.

- Numerically simulate the proposed models, using computational tools to explore the effect of parameter variation and to visualize trajectories, phase portraits, and bifurcation diagrams.

- Critically evaluate existing models (World3, HANDY, Brander-Taylor, etc.) by identifying their assumptions, limitations, domains of validity, and implications for sustainability policies, including trade-offs between economic development, biodiversity conservation, and public health.

- Apply the notions of carrying capacity, resilience, and ecological thresholds to the analysis of case studies such as renewable resource management, habitat conservation, and the effects of inequality on collapse dynamics.

Contents

- MATHEMATICAL FOUNDATIONS

- ORIGINS AND MODELS OF POPULATION AND BIODIVERSITY

- ECONOMY-ENVIRONMENT INTEGRATION

Detailed program

PART 1: MATHEMATICAL FOUNDATIONS

- Differential Equations
  Definitions and classifications: linear and non-linear ODEs, first-order and systems
  Geometric interpretation: direction fields and phase portraits
  Analytical solution for first-order linear ODEs
  The concept of equilibrium point and stability

- Difference Equations
  Definition and interpretation of discrete-time models
  Geometric growth and the logistic map as an introductory example
  Stability of fixed points in one dimension
  Bifurcations: the concept of regime change

- Tools for Dynamical Analysis
  Systems of differential equations: equilibrium points
  Linear stability analysis: eigenvalues and their interpretation
  Limit cycles and bifurcations: transitions between qualitatively different behaviors

- Deterministic chaos

PART 2: ORIGINS AND MODELS OF POPULATION AND BIODIVERSITY

- The Malthusian Model (1798)
  Unlimited Growth

Exponential model
Analysis: Exponential solution, doubling time, explosive growth, Absence of endogenous brakes
The discrete version

- The Verhulst Model (1838)
  Carrying Capacity
  Analysis: Equilibrium points, stability of K, sigmoid growth
  The discrete version: Logistic map
  Period-doubling cascade, chaos

- The Gompertz Model (1825)
  Asymmetry
  Variable growth rate
  The discrete version
  Bifurcations chaos

- Lotka-Volterra Model (1920-1926)
  Predator-Prey Interaction
  Analysis: Limit cycles, equilibrium point, oscillatory dynamics

- MacArthur and Wilson Model (1967)
  Species-Area Relationship
  Habitat Fragmentation

- Levins Model (1969)
  Metapopulation models
  Spatial Models for biodiversity
  Dynamical analysis
  Complex behaviors

PART 3: ECONOMY-ENVIRONMENT INTEGRATION

- The World3 Model (1972)
  The Limits to Growth on a Global Scale

- Forrester and Meadows system dynamics model
  Interaction between population, industrial production, pollution and resources
  Collapse vs. stabilization

- The Brander and Taylor Model (1998)
  Easter Island
  Predator-prey model with economic choice
  Labor allocation between subsistence and "prestige" production

- The Pezzey and Anderies Model (2001)
  Extension of the Taylor Model
  Introduction of a minimum subsistence requirement
  Conservation policies (taxes, quotas)

- The HANDY Model (Human And Nature DYnamics) (2014)
  3-variable model: Population, Wealth, Natural Resources
  Inequality
  Equilibrium points (sustainable, collapse),
  Bifurcations (Hopf, transcritical)

Prerequisites

These are the minimum prerequisites without which the course cannot be taken; students must arrive having already acquired this knowledge. For some of the linear algebra topics, an initial review module is planned.

- Elementary Functions
  In-depth knowledge of exponentials, logarithms, and trigonometric functions.

- Equations and Functions
  Solving equations: linear, quadratic, exponential, and logarithmic equations.
  Function analysis: computing limits, asymptotes, maximum/minimum points.

- Differential and Integral Calculus
  Derivatives: concept of derivative as a rate of change, differentiation rules (product, quotient, chain rule).
  Integrals: integration of elementary functions, integration by parts, substitution.

- Basic Linear Algebra
  Vectors and matrices: fundamental operations (addition, multiplication, inverse).
  Determinants and eigenvalues: concept of eigenvalue and eigenvector, computation of the determinant for matrices.
  Linear systems: solving systems of linear equations.

- Equations and Functions
  Solving equations: linear, quadratic, exponential, and logarithmic equations.
  Function analysis: computing limits, asymptotes, maximum/minimum points.

Teaching methods

Lectures and in-class exercise sessions; all 32 hours of lectures and exercise sessions are of a lecture-based format, to be delivered over 16 meetings.

Assessment methods

Final assessment consists of a written exam, which may be complemented by an optional oral exam.
In the written exam, students are required to solve exercises and may also be asked to answer open-ended questions. The exercises concern the models presented during the course.
In the oral exam, students must be able to discuss all topics covered in the course, including both theoretical and practical aspects. The optional oral exam may affect the final grade either positively or negatively.

Textbooks and Reading Materials

Course slides and teaching materials, including exercises and solutions, will be provided on the e-learning platform. For further details and for the original sources of the mathematical methods and models considered, please refer to the following:

MATHEMATICAL FOUDATIONS

- Hirsch, M. W., Smale, S., & Devaney, R. L. (2004). Differential Equations, Dynamical Systems, and an Introduction to Chaos (2nd ed.). Elsevier Academic Press.
- Brauer, F., & Castillo-Chavez, C. (2012). Mathematical Models in Population Biology and Epidemiology (2nd ed.). Springer.
- Elaydi, S. N. (2005). An Introduction to Difference Equations (3rd ed.). Springer.

ORIGINS AND MODELS OF POPULATION AND BIODIVERSITY

- Malthus, T. R. (1798). An Essay on the Principle of Population. London: J. Johnson.
- Verhulst, P. F. (1838). Notice sur la loi que la population suit dans son accroissement. Correspondance Mathématique et Physique, 10, 113-121.
- Gompertz, B. (1825). On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. Philosophical Transactions of the Royal Society of London, 115, 513-583.
- Lotka, A. J. (1925). Elements of Physical Biology. Baltimore: Williams & Wilkins.
- Volterra, V. (1926). Fluctuations in the abundance of a species considered mathematically. Nature, 118, 558-560.
- MacArthur, R. H., & Wilson, E. O. (1967). The Theory of Island Biogeography. Princeton University Press.
- Levins, R. (1969). Some demographic and genetic consequences of environmental heterogeneity for biological control. Bulletin of the Entomological Society of America, 15, 237-240.

ECONOMY-ENVIRONMENT INTEGRATION

- Forrester, J. W. (1971). World Dynamics. Wright-Allen Press.
- Meadows, D. H., Meadows, D. L., Randers, J., & Behrens, W. W. (1972). The Limits to Growth. Universe Books.
- Brander, J. A., & Taylor, M. S. (1998). The simple economics of Easter Island: A Ricardo-Malthus model of renewable resource use. American Economic Review, 88(1), 119-138.
- Pezzey, J. C. V., & Anderies, J. M. (2001). The effect of subsistence requirements on the collapse of a resource-dependent society. Journal of Economic Behavior & Organization, 45(2), 145- 165.
- Motesharrei, S., Rivas, J., & Kalnay, E. (2014). Human and nature dynamics (HANDY): Modeling inequality and use of resources in the collapse or sustainability of societies. Ecological
- Economics, 101, 90-102.

Semester

First semester

Teaching language

English

Sustainable Development Goals

QUALITY EDUCATION