UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Actuarial Mathematics

2627-2-F1602M022

Learning objectives

The central theme of the course is the set of mathematical and statistical methods used to answer questions such as determining the premium of a life insurance policy or calculating the amount needed to secure a supplementary pension. These are fundamental questions both for the investor purchasing protection (which often has features very similar to those of an investment), and for the insurance companies selling it, which—while collecting premiums—must monitor their exposure to various sources of risk in a continuously evolving regulatory environment.

The course consists of two modules: Actuarial Mathematics (taught by Prof. Fabio Bellini) and Statistics for Insurance (taught by an industry professional). We recommend including this course in your study plan alongside Insurance Risks, held in the second semester, which focuses more on methods for non-life insurance.

In summary, we outline the following learning objectives:

- Understand the fundamental concepts of modeling human life duration and use them to calculate the actuarial value and premium of the main types of life contingencies
- Use R to download, visualize, and model mortality data, as well as to perform actuarial calculations and simulations
- Understand the quantitative aspects of insurance regulation, with particular focus on the Solvency II framework as it applies to life insurance

Contents

1) The modelling of human lifetime duration

2) Formulas for the calculation of actuarial present values and premium payment schemes in life contingencies

3) The mathematical reserve and the profit decomposition in traditional actuarial mathematics

4) Introduction to Solvency II and to risk management in insurance companies

5) Health insurance policies and an introduction to multi-state models

6) Reinsurance contracts and an introduction to pricing methodologies

Detailed program

1. **Modelling human lifetime**

Mortality tables, life and death probabilities, deferred death probabilities. Identities related to life and death probabilities. Other one-year measures of mortality: central death rate and mortality odds. The probabilistic model: the complete and curtate future lifetime random variable, survival function, and conditional survival function. Calculation of complete and curtate life expectancy. Force of mortality and mortality laws: Gompertz, Makeham, and Heligman and Pollard. Heterogeneity and introduction to risk aggravation. Introduction to stochastic mortality. The Lee-Carter model.

Things to be able to do in R:

- download, import, and visualize mortality tables from the ISTAT website and the Human Mortality Database
- create lifetable objects using the *lifecontingencies* package and use the functions pxt, qxt, exn, rLife
- estimate, visualize outputs and residuals, and forecast mortality using the Lee-Carter model and the *demography* package
- construct a mortality table starting from a mortality law, as in the Heligman and Pollard example, using the *MortalityLaws* package

2. **Formulas for calculating actuarial values and premium payment schemes for life contingencies**

Definition of the actuarial value of a *life contingency*. Main types of benefits in life insurance: pure endowment, term insurance, whole life insurance, temporary and whole life annuities. Formulas for calculating actuarial values and standard actuarial notation: nEx, nAx, Ax, nax, ax. Recursive formulas for actuarial values. Determination of the fair premium: the actuarial equivalence principle. Premium payment schemes: single initial premium, level annual premiums, natural premiums, recurring single premiums.

Things to be able to do in R:

- create *lifetable* and *actuarialtable* objects in the *lifecontingencies* package, starting either from empirical data or from mortality laws
- calculate present values and accumulated values using presentValue, annuity, and accumulatedValue
- calculate actuarial values and fair premiums using Axn, Exn, axn

3. **Mathematical reserve and profit decomposition in traditional actuarial mathematics**

Definition of the prospective net mathematical reserve as the difference between the actuarial value of future benefits still to be paid and the actuarial value of future premiums still to be received. Time evolution of the reserve in the case of term insurance, pure endowment, and endowment insurance. Fouret's equation for endowment insurance. Decomposition of the premium into risk premium and savings premium. Homans' contribution formula.

Things to be able to do in R:

- build an R function to calculate the time evolution of the reserve of an endowment insurance contract by applying the definition
- build an R function to calculate the time evolution of the reserve of an endowment insurance contract using Fouret's equation
- build an R function to calculate natural premiums, risk premiums, and savings premiums for an endowment insurance contract

4. **Introduction to Solvency II and risk management in insurance companies**

Basic principles of Solvency II. Keywords: Best Estimate, Risk Margin, Solvency Capital Requirement, Own Funds. Approaches to SCR calculation: standard formula, undertaking-specific parameters, internal models.

5. **Health insurance policies and introduction to multi-state models**

Types of health insurance policies and coverages. Introduction to the collective risk model, not required for the exam and to be studied in more depth in Actuarial Statistics. Multi-state models.

Things to be able to do in R:

- simulation and pricing of multi-state policies using the *markovchain* package

6. **Reinsurance contracts and introduction to pricing methodologies**

Reinsurance contracts. Pricing methodologies, not required for the exam and to be studied in more depth in Insurance Risks.

7. **Basic R concepts**

- Basic concepts of the RStudio working environment
- fundamental data types: vectors, lists, data frames
- NA, NULL, and NaN
- basic concepts of data visualization in base R: hist and plot
- simulation of random numbers
- construction of loops
- construction of functions
- lifetable and actuarialtable objects and associated functions

Prerequisites

Basic knowledge of general mathematics (derivatives and integrals), financial mathematics (annuities and present values), and probability (random variables).

Teaching methods

Lectures, bring-your-own-device R programming

Assessment methods

Oral exam and discussion of an R project.

Textbooks and Reading Materials

Testi di riferimento per l'esame (vedere le sezioni nel programma dettagliato)

- **OP** Olivieri, A., Pitacco, E., Introduction to Insurance Mathematics
- **PDHO** Pitacco, E., Denuit, M., Haberman, S., Olivieri, A. Modelling Longevity Dynamics for Pensions and Annuity Business
- **CV** Cesari, R., Valerio A. Risk Management e imprese di assicurazione

Testi di approfondimento

- **D** Davies, T. The book of R: a first course in programming and statistics
- **P1** Pitacco, E. Health insurance: basic actuarial models
- **P2** Pitacco, E. ERM and QRM in Life Insurance
- **AI** Wutrich et al. AI tools for actuaries

Semester

First semester

Teaching language

Italian

Sustainable Development Goals

QUALITY EDUCATION