UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Advanced Foundations of Mathematics for AI

2627-1-F9103Q001

Aims

The aim of this course is to provide the mathematical foundations for the optimization and data manipulation algorithms, which are widely employed in the artificial intelligence domain. A number of applications are provided, so that the student will be able to tackle and solve constrained and unconstrained optimization problems, analyze data through dimensionality reduction and Fourier transforms, while the theoretical part will constitute a solid background for understanding and mastering related, more recent techniques, which are constantly developed in the field.

In terms of Dublin descriptors, the aims are:

1 (Knowledge and understanding) The student will acquire knowledgeof tools of linear algebra and of Fourier analysis, the notion of optimization and the basics of convex analysis and convex optimization.

2 (Applying knowledge and understanding) The student will be able to apply the methods of linear algebra entering the most used algorithmsand in dimensionality reduction techniques, and to set a problem of constrained optimization, paying attention to the convexity properties.

3 (Making judgements) The student must be able to inderstand and critically evaluate definitions, statements and proofs, besides recognising the tools which are better suited to solve the proposed problems.

4 (Communication skills) An oral examination is required, in order to make sure that every student, whatever his previous studies are, is capable of describing with clarity and rigour the fundamental concepts, correctly using the mathematical language.

5 (Learning skills) The student will be able to learn on his own methods of linear algebra, related to the considered ones, and to broaden the knowledgeof dimensionality reduction techniques and convex optimization methods.

Contents

The course consists of a theoretical part and a part of exercises. The theoretical part will start by recalling the basic concepts of linear algebra and multivariate calculus which are needed, then will cover the topics of optimization (in particular, convex optimization), dimensionality reduction techniques and Fourier analysis. In the exercises examples of related problems and applications are given.

Detailed program

- Linear Algebra: Eigenvalues, eigenvectors, diagonalization and spectral theorem. Positive definite matrices, singular value decomposition.
- Fourier transform and series: Fourier series for periodic functions, Fourier transform of continuous and discrete signals. Definitions and basic properties, inversion and differentiation, convolutions.
- Multivariate calculus: Partial derivatives, differential, Jacobian matrix, Hessian matrix, Taylor's theorem.
- Optimization: Unconstrained critical points and characterization through Hessian matrix. Gradient descent, Newton's method. Implicit function theorem, constrained critical points, Lagrange multipliers.
- Convex optimization: Convex sets, convex functions. Convex conjugates.
- Convex optimization problems: definition, notable cases, duality, strong duality and optimality condition.
- Linear and nonlinear dimensionality reduction techniques: Linear projectors, principal component analysis, independent component analysis, kernel principal component analysis.

Prerequisites

Foundations of calculus: derivatives, integrals, numerical series. Foundations of linear algebra: vector spaces and linear applications, matrix representation.

Teaching form

56 hours of lectures and assisted exercises, in each lecture both a teaching and an exercise part can occur. The teaching method will be lecture based and in person, the attendance is highly recommended.

Textbook and teaching resource

M. P. Deisenroth, A. A. Faisal, C. S. Ong, Mathematics for Machine Learning, Cambridge University Press (2020). Lecture notes.

Semester

First.

Assessment method

The exam is individual and consists of a written and an oral part, both of them are required. In the written exam, the proficiency in applying mathematical notions to the solution of exercises and problems is evaluated. As an alternative to the written part only, two partial written exams are planned. The oral exam is focused, instead, on assessing the knowledge of the mathematical notions and the ability of expressing them in an adequate way, as well as on establishing the understanding of the deduction processes which link the mathematical objects.

Office hours

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