UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Dynamical Systems and Classical Mechanics

2627-2-E3502Q010

Aims

The course aims to present the fundamental ideas of Classical Mechanics, from the formulation of Galileo and Newton to the Lagrangian and Hamiltonian formulations, and to provide the mathematical tools required for their understanding and application to the study of dynamical systems.

The expected learning outcomes are described according to the Dublin Descriptors.

Knowledge and understanding

At the end of the course, students are expected to know and understand:

the basic definitions, principles and fundamental statements of the different formulations of Classical Mechanics;
the relation between the Newtonian, Lagrangian and Hamiltonian formulations;
the elementary qualitative tools for the study of dynamical systems, with particular reference to equilibrium points, stability, linearization and one-degree-of-freedom systems;
some fundamental examples, such as isolated systems, the harmonic oscillator, the two-body problem, the Kepler problem, rigid bodies and the Lagrange top;
the role of symmetries and conservation laws in the reduction and analysis of mechanical systems.
Applying knowledge and understanding

At the end of the course, students are expected to be able to:

derive the equations of motion of simple mechanical systems, both in the Lagrangian and in the Hamiltonian formulation;
choose suitable generalized coordinates for the description of a constrained system;
apply D'Alembert's principle and the Euler-Lagrange equations to systems with holonomic constraints;
use symmetries and conserved quantities to reduce the number of degrees of freedom;
discuss the motion qualitatively in significant examples, also by means of energy level curves and phase portraits;
reduce, in simple cases, the solution of the equations of motion to quadratures;

solve exercises and problems requiring the combined use of analytical, geometrical and mechanical tools.
Making judgements

The course aims to develop the ability to critically analyse definitions, statements and proofs; to recognize the validity of a mathematical argument and its interpretation within the mechanical model under consideration; and to select independently the most appropriate method for the problem at hand.

These skills will be developed through the discussion of examples, the comparison between different formulations of the same problem, the guided solution of exercises and the analysis of the assumptions required for the correct application of the theoretical results.

Communication skills

Students are expected to acquire the ability to discuss mathematical models of dynamical systems clearly and rigorously, both orally and in writing. They should also be able to present a proof in a coherent and understandable way, justify the main steps of a solution and use mathematical and mechanical terminology appropriately. When relevant, they should also be able to translate the mathematical content of a result into a qualitative description of the corresponding mechanical phenomenon.

Learning skills

The course aims to provide students with the conceptual and technical tools needed to continue studying the dynamics of classical systems independently and at a more advanced level. At the end of the course, students should be able to approach new topics with method and rigour, using previous knowledge, lecture notes, textbooks and other teaching material to deepen and update their skills. The interdisciplinary nature of the course, in which analytical and geometrical tools are used to study both abstract and concrete dynamical systems, contributes to building a solid theoretical basis for the degree programme in Mathematics.

Contents

Review of Newtonian mechanics. Ordinary differential equations and qualitative study of dynamical systems. D’Alembert’s principle and Lagrangian mechanics. The two-body problem and Kepler’s laws. Rigid body mechanics. Hamiltonian mechanics. Symmetries, conservation laws and canonical transformations.

Detailed program

Review of differential equations and dynamical systems

Vector fields and systems of first-order differential equations. Autonomous systems. Equilibrium points and stability. Lyapunov theorem. Linearization of nonlinear systems near an equilibrium point. One-degree-of-freedom systems. Energy level curves and qualitative study of motion in relevant examples.

Lagrangian mechanics

Euler-Lagrange equations. A point particle constrained to a regular curve. A point particle constrained to a regular surface. Holonomic constraints and D’Alembert’s principle. Equilibrium points and small oscillations. Variational formulation of the Euler-Lagrange equations. One-parameter groups of diffeomorphisms and symmetries. Noether’s theorem. The two-body problem. Kepler’s laws.

Rigid body

The group of rotations in three-dimensional space and angular velocity. Inertial and non-inertial reference frames. Rigid body mechanics. Inertia operator. König's theorem. Euler equations for the rigid body. Euler angles. Lagrange top.

Hamiltonian mechanics

Legendre transform. Hamilton's equations. Poisson brackets and their properties. Lie algebra structure associated with Poisson brackets. Symmetries and conservation laws in Hamiltonian mechanics. Variational formulation of Hamilton's equations. Canonical transformations and equivalent conditions for canonicity. Liouville's theorem.

Prerequisites

Basic knowledge of Mathematical Analysis I and II, Linear Algebra and Geometry, and Physics I is required. In particular, familiarity with differential calculus in one and several variables, basic integral calculus, elementary linear algebra, analytic geometry, basic ordinary differential equations and the fundamental notions of Newtonian mechanics is assumed

Teaching form

The course consists of 112 hours, corresponding to 12 ECTS credits, and is taught in Italian.

Teaching activities are organized as follows:

64 hours of in-person lectures, mainly in delivery mode, devoted to the presentation of concepts, theoretical results, proofs and fundamental examples;
48 hours of in-person exercise sessions, in erogative mode, devoted to the guided solution of exercises and problems, the discussion of solution strategies and the comparison of alternative methods.

No remote teaching hours are planned.

During the lectures, the instructor will present the theoretical contents, emphasizing motivations, connections and possible conceptual difficulties. Questions and short discussions will also be proposed in order to encourage active participation and to check the progressive understanding of the topics. The exercise sessions will be devoted to the application of the theoretical tools to concrete problems and to preparation for the written exam.

Textbook and teaching resource

Recommended textbooks:

V. I. Arnold, Mathematical Methods of Classical Mechanics, Springer.
A. Fasano, S. Marmi, Meccanica Analitica, Bollati Boringhieri, 2002.
L. D. Landau, E. M. Lifshitz, Mechanics, Pergamon Press.
N. M. J. Woodhouse, Introduction to Analytical Dynamics, Oxford Science Publications, Clarendon Press, Oxford University Press, New York, 1987.

G. Dell’Antonio, Elementi di Meccanica, Liguori, Napoli, 1996.

Additional material, including possible lecture notes by the instructor, will be made available through the course e-learning platform.

For examples and exercises, the following texts are also recommended:

F. Talamucci, Esercizi svolti sul formalismo lagrangiano e hamiltoniano con brevi richiami di teoria, Edizioni Nuova Cultura, 2014.
A. Celletti, Esercizi e Complementi di Meccanica Razionale, Aracne Editrice, 2003.
G. Benettin, Eserciziario per il corso di Fisica Matematica, Padova, 2017, freely available from the author’s webpage.

Semester

Second semester

Assessment method

The exam is individual and is aimed at assessing both the ability to solve exercises and problems and the knowledge and understanding of the theoretical contents of the course.

The final exam consists of a written test and, as a rule, an oral exam. In the cases specified below, the written test may be considered sufficient for the final grade, subject to evaluation by the instructor.

Written test

The written test consists of exercises and problems. It typically includes one exercise in Lagrangian mechanics and one exercise in Hamiltonian mechanics, or problems requiring the integrated use of tools presented during the course. The duration of the written test is three hours.

The written test assesses in particular:

the ability to set up a mechanical problem correctly;
the ability to choose suitable coordinates and variables;
the ability to derive the equations of motion;
the correctness of computations and arguments;
the ability to interpret qualitatively the result obtained;
the clarity and completeness of the written exposition.

Correct answers without adequate explanations, motivations or intermediate steps will not receive full marks. Admission to the oral exam requires a mark in the written test of at least 15/30.

Oral exam

The oral exam consists of an interview on the topics covered in the lectures. It requires knowledge of the definitions, statements and proofs of the main results of the course, as well as the ability to illustrate their meaning through relevant examples.

The oral exam assesses in particular:

knowledge of the theoretical contents of the course;
understanding of the mathematical and mechanical meaning of the results;
the ability to present proofs clearly and coherently;
the ability to connect different topics of the course;
the appropriate use of disciplinary language;
theoretical awareness in the interpretation of exercises and models.

Relation between written test and oral exam

If the written test receives a mark of at least 18/30, it may be considered sufficient for the final outcome of the exam and may allow the student to waive the oral exam, subject in any case to evaluation by the instructor. The instructor may still require the oral exam whenever further assessment of the student's preparation is considered necessary. If the oral exam is not taken, the final mark cannot exceed 25/30, regardless of the mark obtained in the written test.

The oral exam must be taken in the same exam session in which the written test was taken with the exception of June and July when the oral exam can be postponed respectively to July or September.

If the oral exam is also taken, the final grade takes into account the results of both tests. The written test mainly assesses application skills and problem-solving ability; the oral exam mainly assesses theoretical knowledge, argumentative ability and conceptual mastery.

Midterm tests

During the course, two written midterm tests will be offered, concerning respectively the first and the second part of the programme. The midterm tests are individual and consist of exercises and problems similar, in structure and aims, to those of the final written test.

The midterm tests assess the progressive understanding of the topics covered, the ability to apply the tools introduced in lectures and exercise sessions, the correctness of the solution procedure and the clarity of the written exposition.

Students who pass both midterm tests with a mark of at least 15/30 are admitted to the possible oral exam. In this case, the oral exam must be taken no later than the July exam session.

If the midterm tests obtain an average mark of at least 18/30, they may be considered sufficient for the final outcome of the exam and may allow the student to waive the oral exam, subject in any case to evaluation by the instructor. The instructor may still require the oral exam. If the oral exam is not taken, the final mark cannot exceed 25/30, regardless of the mark obtained in the written tests.

Assessment criteria and grading scale

The overall assessment takes into account technical correctness, conceptual understanding, the ability to apply the methods of the course, clarity of exposition and mastery of mathematical and mechanical language.

The indicative grading scale is as follows:

18-19: preparation limited to a small number of topics in the programme; partial ability to discuss and analyse the material; expository skills and terminology not always correct; limited capacity for independent elaboration.

20-23: sufficient preparation on a significant part of the programme; mainly procedural application skills; generally correct terminology, although not always precise; sometimes uncertain exposition.

24-27: good preparation on a broad range of topics covered in the course; ability to develop arguments and analyses independently; good ability to apply knowledge to exercises and problems; correct use of disciplinary language.

28-30 cum laude: complete and thorough preparation on the topics of the programme; full mastery of techniques and theoretical contents; ability to discuss topics independently and critically; clear and rigorous exposition; ability to connect different topics of the course and, where appropriate, relate them to other areas of mathematics and physics.

Office hours

By appointment, to be arranged by e-mail. Office hours may take place in person or remotely through the University platform.

Sustainable Development Goals

QUALITY EDUCATION