UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Introduction To Numerical Analysis

2627-2-E3502Q012

Aims

The aim of this course is to present the basic topics of Numerical Analysis, both from the theoretical and algorithmical point of view, that every Mathematician should know.

The learning outcomes are:

Knowledge:

- Knowledge and understanding of the fundamental techniques of Numerical Analysis, including theorems and proofs;
- Knowledge and understanding of the scientific problems related to numerical analysis.

Skills:

- Being able to translate the theoretical results into algorithms;
- Choose the right numerical method to solve a given problem;
- Critical analysis of the results;
- Being able to present in a clear and precise way the theoretical results shown in the course and their practical applications.

Dublin Descriptors

Knowledge and Understanding
The student demonstrates knowledge of the fundamental principles of numerical analysis, including methods for numerical solution of equations, interpolation, numerical integration, and function approximation.

Applying Knowledge and Understanding
The student is able to select and apply appropriate numerical methods to solve basic mathematical problems, analyze the results, and evaluate the accuracy and effectiveness of the methods used.

Making Judgements
The student is capable of critically evaluating the validity of the obtained numerical solutions, understanding the limitations of numerical methods and the sensitivity of algorithms to input data.

Communication Skills
The student is able to clearly and coherently present the obtained results using appropriate technical language, and to properly document the numerical procedures adopted.

Learning Skills
The student develops the ability to independently deepen knowledge of further numerical methods by using bibliographic resources and software tools, in order to improve and broaden their understanding of numerical analysis.

Contents

The main topics are:

- Floating Point Arithmetic
- Numerical Linear Algebra: linear systems, eigenvalues computation
- Root-finding algorithms
- Polinomial Interpolation
- Least squares method and QR decomposition
- Quadrature Formulas

Detailed program

- **Floating Point Arithmetic:** Representation of real numbers, Representable numbers, Approximation of real numbers on a computer, Floating point arithmetic, Rounding to even, Computation of elementary functions;
- **Gaussian elimination and the decomposition PA=LU:** Linear systems, Gaussian elimination, Decomposition PA=LU;
- **Topics in linear algebra:** Scalar products and norms, Norms in R^n, Matrix norms;
- **Stability of the Gaussian elimination:** Perturbation of a linear system, Application to the Gaussian elimination;
- **Cholesky decomposition:** Symmetric and positive definite matrices, Cholesky decomposition, Application to the solution of a linear system;
- **Iterative methods for linear systems:** Motivations, Iterative methods for systems of linear equations, Stopping criteria;
- **Eigenvalues:** Gershgorin circles, Perturbation analysis of eigenvalues, Power method;
- **Root-finding:** Bisection method, Newton methods and variations, Experimental measurement of the order of convergence, Brent method, MATLAB implementation;
- **Polynomial interpolation:** Weierstrass theorem, Interpolation, Analysis of interpolation algorithms, Conditioning, Interpolation of functions;
- **Splines;**
- **Least squares and QR factorization:** Overdetermined linear systems, Geometrical interpretation, QR decomposition, Linear regression, Using QR factorization to solve linear systems;
- **Quadrature formulas:** Interpolatory quadrature formulas, Trapezoidal rule, Simpson rule, Newton-Cotes formulas, Adaptive quadrature formulas.

Prerequisites

First year courses *Analisi 1* and *Algebra Lineare e Geometria.*

Teaching form

44 hours of in-person, lecture-based teaching (5.5 ECTS)
20 hours of online, lecture-based teaching (2.5 ECTS)

48 hours of in-person, exercises classes teaching (4 ECTS)

The course is taught in italian.

Textbook and teaching resource

Teacher's notes and recordings of the lectures are available on the web site of the course.

Semester

1?? semester.

Assessment method

Practice test on the computer followed by oral examination. Final mark out of 30.

In the first part of the examination the student will be assigned a few problems in numerical analysis to be solved on the computer using the MATLAB codes developed during the course.

The second part is a standard oral examination which requires the exposition of statements and proofs of the theorems, definitions, examples/counterexamples and computational techniques. The practice test will also be discussed.

In order to take the oral examination, the students need to pass the practice test with a mark of at least 18 out of 30. The final mark will take into account both tests.

Both the practical and theoretical/oral exams must be taken during the same exam session.

Office hours

On appointment.

Sustainable Development Goals

QUALITY EDUCATION