UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Algebra I

2627-1-E3502Q001

Aims

1. Knowledge and understanding
   At the end of the course, students will have acquired a solid understanding of the fundamental algebraic structures (groups, rings, fields) and the core methods of abstract algebra. They will be able to recognize, describe, and compare these structures, understanding their properties and interrelations.

2. Applying knowledge and understanding
   Students will be able to apply the theoretical concepts learned to solve algebraic problems, both through routine exercises and in more complex situations requiring autonomous reasoning. They will be capable of using algebraic methods to formalize and address problems in mathematical and, where appropriate, interdisciplinary contexts.

3. Making judgements
   Students will develop the ability to critically analyze algebraic problems, evaluate different solution strategies, and choose the most appropriate tools. They will be able to formulate conjectures and support their validity through rigorous logical reasoning.

4. Communication skills
   Students will be able to clearly and rigorously express concepts, definitions, theorems, and proofs, using the formal language of algebra. They will also be able to discuss solutions and strategies with peers and instructors, both orally and in writing.

5. Learning skills
   The course will equip students with the conceptual and methodological tools necessary to pursue further studies in algebra and related mathematical areas independently. Students will be encouraged to adopt an active approach to learning, through problem-solving, consulting advanced texts, and reflecting on the meaning and application of algebraic concepts.

Contents

Greatest Common Divisor (GCD) and the Euclidean Algorithm for integers and polynomials. Congruence modulo n. Elementary Group Theory. Group Actions. Sylow Theorems.

Detailed program

Euclidean division for integers and polynomials. Greatest Common Divisor (GCD) and the Euclidean Algorithm for integers and polynomials. Linear Diophantine equations in Z. Prime and irreducible elements. Prime if and only if irreducible in Z. Fundamental Theorem of Arithmetic. Euclid's Theorem on prime numbers.

Congruence modulo n. Residue classes. Quotient ring Z/nZ. Linear congruences and the Chinese Remainder Theorem in Z.

Elementary Group Theory. Binary operations. Definition of a group and notable examples. Subgroups. Notable examples. Subgroup criterion. Subgroup generated by a subset. Subgroups of Z. Cyclic groups. Order of an element. A finite cyclic group has exactly one subgroup of order d for each divisor d of the order of the group. Euler's totient function. Generators of a finite cyclic group. Permutations, cycles, and decomposition of a permutation into a product of disjoint cycles. Order of a permutation. Parity of a permutation. The alternating group. Lagrange's Theorem. Fermat's Little Theorem. Normal subgroups and quotient groups. Homomorphisms. Kernel and image. Isomorphism theorems. Internal and external direct products of groups. Characterization of direct products.

Group actions. Orbits and stabilizers. Orbit–stabilizer theorem. Notable examples of group actions (regular representation, conjugation on elements and subgroups, normalizer of a subgroup). Cayley's Theorem. Dihedral groups. Burnside's Lemma. Necklaces.

Sylow Theorems. Applications to the simplicity of finite groups.

Prerequisites

Basic notions of high school algebra and analysis

Teaching form

The course is organized in Lectures (48 hours, 6 CFU) and Exercise classes (24hours, 2CFU). Definitions, results and relevant theorems will be presented in Lectures, providing examples and problems making use of the notions introduced.

Textbook and teaching resource

Textbook
I. N. Herstein, Algebra, Editori Riuniti
Written notes on some topics available on this platform.

Suggested readings:
Jacobson, Basic Algebra I, Dover, 1985

Semester

Second semester

Assessment method

Written exam, and optional oral exam, only if requested either by the teacher or a student. Assessment graded on a 30-point scale.

The written examination evaluates the knowledge of the course contents and the ability to apply them to problem solving. The correctness of the answers, the mathematical language as well as the rigor and clarity of the exposition will be evaluated. Two partial tests, during the course, allow you to take the oral test.

The oral examination consists of an interview on the course contents and may be requested by a student only after achieving a mark of at least 18/30 in a written exam. To register a grade higher than 27/30, it is mandatory to also take the oral exam.

During the year there are 6 exam sessions and 2 ongoing tests.

Office hours

By appointment via e-mail.

Sustainable Development Goals

QUALITY EDUCATION | GENDER EQUALITY