UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Classical Mechanics

2627-2-E3005Q013

Aims

The content of the Course presents the basic ideas of Classical mechanics, from the Galileo-Newton formulation to those of Lagrange, Hamilton and Jacobi. The necessary mathematical tools for a proper comprehenson of these fundamental theories will be introduced and discussed.

The student, by the end of the course, will have acquired the following skills:

1. Knowledge and understanding
   The student will have acquired a solid but foundational understanding of the Lagrangian and Hamiltonian formalisms of classical mechanics. They will be able to recognize the conceptual meaning of symmetries, constraints, and conservation laws, and understand the role of these tools in the theoretical formulation of physics.

2. Applying knowledge and understanding
   The student will be able to apply the methods learned to analyze and solve mechanical problems, both in simple systems and in the presence of constraints. They will also be capable of reformulating a physical problem using the tools of Analytical Mechanics and applying appropriate mathematical techniques.

3. Making judgements
   The student will be able to critically assess methodological choices in solving a physical problem and identify the most appropriate formalism for modeling it, while distinguishing between various simplifying assumptions.

4. Communication skills
   The student will be able to clearly and rigorously present the main concepts of Analytical Mechanics, using the correct language of physics and mathematics. They will be able to structure and explain the solution to a problem, justifying each step taken.

5. Learning skills

The course will provide the student with the conceptual and methodological foundations needed to successfully approach more advanced courses in theoretical physics and applied mathematics, as well as to independently deepen their understanding of more complex topics in classical mechanics.

Contents

Newtonian Mechanics (a reminder).

Ordinary differential equations (dynamical systems). Qualitative analysis.

Lagrangian Mechanics.

Hamiltonian mechanics.

Detailed program

1) Space-time and events. Newton's principia and the dynamics of point masses.

2) Dynamical systems as mathematical models for physical phenomena. Dynamical Systems Phase diagrams of conservative Newtonian systems in one dimension. The Lotka-Volterra system and Volterra’s laws. Bifurcation diagrams. Linearization, stability and the theorems of Lyapunov (statement).

3) Dynamics of systems of point masses.

4) Constraints, degrees of freedom, and free coordinates. The D'Alembert principle and Lagrangian Mechanics.

5) The Lagrangian and the Euler-Lagrange equations. Variational principles. Central motions and the Kepler problem. Lagrangian formulation of the Lorentz force. Theory of small oscillations. Further applications. Noether’s theorem. Basic notions of the theory of rigid bodies. Applications: rigid bodies in the plane. The Lagrange top.

6) Hamiltonian Mechanics: Hamilton equations and their variatiional formulation. Canonical transformations. Canonical contact (point) transformations. Poisson brackets and constants of the motion. Infinitesimal canonical transformations and Noether’s theorem in Hamiltonian Mechanics. Hamiltonian flows and phase space volume invariance.

7) The Hamilton-Jacobi equation. Complete integrals. Separation of variables. Completely integrable systems, action-angle variables and Liouville's theorem (statement). Perturbation theory. The average principle. Perturbation of isochronous systems. The KAM theorem (statement).

Prerequisites

The content of the courses of Calculus I, Linear Algebra and Geometry, Physics I.

Teaching form

- Lectures (5 CFU) via expository teaching. Students will attend lectures where the instructor will present theoretical material and demonstrate problem-solving techniques.

- Classes (3 CFU) via mixed expository and interactive teaching, In expository sessions, students will attend expository-tipe classes where the instructor will apply the theoretical apparatus exposed in the lectures to solve problems in classical mechanics. Besides these expository classes, the course will incorporate interactive teaching methods. This will involve group activities, discussions, and hands-on problem-solving sessions to enhance understanding and foster active participation. We expect that some 10 of classes will be delivered in the interactive way.

Videorecordings of lectures and classes will be available online.

Textbook and teaching resource

References:

L.D. Landau, E.M. Lifshits, “Course of Theoretical Physics, Vol. I: Mechanics” (Pergamon)

H Goldstein, C. Poole, J. Safko, “Classical Mechanics”.

Lecture Notes available on the e-learning page.

Notes of (some of) the lectures, available on the e-learning page.

Semester

Second semester

Assessment method

Written exam (with possible oral exam). The written exam involves the discussion and solution of significant problems in Dynamical Systems, Lagrangian Mechanics, and Hamiltonian Mechanics, as well as questions concerning the theoretical aspects of the course.
There will be a partial written exams during the course, around end of april. The written part of the exam will be completed in one of the exam sessions following the end of the course, by the September exam session. Subsequently, candidates will have to (re)take the entire set of exercises.
In case of absence from the partial written exam, or insufficient results, it will be possible to retake the relevant exercise upon first completion of the written part (by September).
The partial written exam focuses on Lagrangian mechanics. The written exam is completed after the end of the course with problems in Hamiltonian mechanics. There will be three exam sessions: Winter session (January-February), Summer Session (June-July), Fall Session (September).

The oral exam involves a discussion of the written paper and the presentation of some fundamental points of the program. The questions will be chosen (by the instructor) from a list that will be communicated to students at the

end of the course. During the oral exam, students may also be asked to address problems similar to those covered during the course.
The exam aims to verify the achievement of points 1), 2), 3), 4), and 5) of the “Objectives” section.
In addition to the ability to solve exercises, the written assessment will also take into account the student's theoretical knowledge of the entire program. If this ability is evident from the written exam, and the grade is at least 22/30, the written exam will be considered sufficient for the overall assessment (without the need for an oral exam). In this case, the final grade will be equal to the grade obtained in the written exam if it is less than or equal to 26/30; 26/30 if higher. Students may request to take the oral exam (in the same exam session) if their written exam score is higher than 12/30.
In the case of a written test with a grade between 12/30 and 21/30, the student must take the oral exam in order to pass the course. The instructor may also require students with a written test grade higher than 21/30 to take the oral exam if they do not demonstrate a clear theoretical understanding of the entire program.

Students may still request to take the oral exam (in the same session), provided that they have obtained an overall written exam grade of at least 12/30. This request must be made within one day of the publication of the written exam results.

If the exam consists of a written and an oral component, the written exam will account for 2/3 of the overall grade.

It will be possible to take the exam in English, upon request to be communicated by email to the teachers at least one week before the written exam.

Office hours

Meetings with individual students or small groups thereof are to be agreed via e-mail or the e-learning page.

Sustainable Development Goals

QUALITY EDUCATION