UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Financial Mathematics M

2627-2-F8206B029

Learning objectives

The course introduces students to the mathematical foundations of continuous-time financial modelling, with particular emphasis on stochastic processes, stochastic integration and their use in the pricing of financial derivatives.
By the end of the course, students are expected to be able to:

1. understand the main concepts of continuous-time stochastic processes used in mathematical finance, including martingales, Brownian motion, quadratic variation and stochastic integrals;
2. apply Ito calculus to solve simple stochastic differential equations and to derive key results used in financial modelling;
3. explain the logic of risk-neutral valuation and the role of equivalent martingale measures in asset pricing;
4. derive and interpret the Black-Scholes equation and the corresponding pricing formula from both PDE and martingale-measure perspectives;
5. recognise the main features and limitations of basic stochastic volatility models, including the Hull-White and Heston frameworks;
6. solve exercises using appropriate mathematical notation, rigorous reasoning and coherent economic interpretation.

Contents

The course covers the mathematical tools required for continuous-time finance and their applications to derivative pricing. Topics include probability foundations, conditional expectation, finite variation processes, martingales, Brownian motion, quadratic variation, Ito integration, Ito formula, change of measure, the Black-Scholes model, the Fundamental Theorem of Asset Pricing, and introductory stochastic volatility models.

Detailed program

1. Probability foundations: probability spaces, random variables, expected value, conditional expectation and basic convergence concepts relevant to continuous-time models.
2. Finite variation processes: definition, main properties and integration with respect to finite variation processes.
3. Martingales and Brownian motion: martingales in discrete and continuous time, Brownian motion, filtration, adapted processes and basic path properties.
4. Quadratic variation: definition and interpretation; quadratic variation of Brownian motion and martingales; relevance for stochastic calculus.
5. Stochastic integration: elementary stochastic integrals, construction of the Ito integral and main properties of stochastic integrals with respect to martingales.
6. Ito formula and stochastic differential equations: Ito formula, exponential martingales and applications to the solution of simple stochastic differential equations.
7. Tanaka formula and integration by parts: extension of Ito calculus, local time intuition and integration-by-parts formula for stochastic processes.
8. Change of measure: Radon-Nikodym derivatives, equivalent probability measures and changes in the characteristics of stochastic processes under a new measure.
9. The Black-Scholes model: assumptions and structure of the model; self-financing portfolios; Black-Scholes PDE; option pricing through the equivalent martingale measure approach.
10. Fundamental Theorem of Asset Pricing: absence of arbitrage, risk-neutral measures and their use in the valuation of contingent claims.
11. Stochastic volatility models: motivations, stochastic components of volatility, market completeness and incompleteness; overview of the Hull-White and Heston models.
12. Selected derivatives and applications: examples of derivative securities and applications of the tools developed in the course to basic pricing problems.

Prerequisites

Students are expected to have a solid background in probability, statistics and mathematical methods. In particular, familiarity with random variables, expectation, conditional probability, basic calculus, linear algebra and elementary differential equations is recommended. Previous exposure to basic financial mathematics is useful but not strictly required.

Teaching methods

Teaching is based on lectures and problem-solving classes for a total of 42 hours. Lectures introduce the theoretical concepts and mathematical results, while classes focus on exercises, applications and the interpretation of continuous-time financial models. Additional online materials, such as lecture notes or exercises, may be used to support individual study.

Assessment methods

Assessment is based on a written examination. The exam includes exercises designed to assess both technical competence and conceptual understanding. Exercises evaluate the students ability to use the mathematical tools introduced in the course, including stochastic integration, Ito formula, change of measure and derivative pricing

techniques. The final grade takes into account correctness of the solutions, clarity of the mathematical reasoning, appropriate use of notation, ability to justify each step, and understanding of the link between the mathematical model and the financial application.

Textbooks and Reading Materials

S. Shreve, Stochastic Calculus for Finance, Springer, 2004.

Lecture Notes

Semester

First semester

Teaching language

Italian. Technical terminology, selected references and some teaching materials may be provided in English.

Sustainable Development Goals

QUALITY EDUCATION