UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Probability and Computational Statistics M

2627-1-F8206B003

Learning objectives

The course consists of two parts:

1. Applied probability
2. Computational statistics

Part 1 aims to provide students with advanced probabilistic tools necessary to understand statistical methodology and to apply it in the economic, business, and financial fields. By the end of the course, students will be able to use probabilistic tools to understand: advanced methodologies for multivariate statistical analysis, advanced methods of statistical inference in both classical and Bayesian frameworks, spatial statistical models, models for high-dimensional data, and complex sampling designs.
Part 2 provides an introduction to the most important computational statistical methods. Students will be introduced to the use of R for the implementation of the computational methods shown during the course.
For additional details, refer to the syllabus of each part of the course.

Contents

PART 1. We start with the different defintions of Probability, and then we move to introduce the axiomatic definition of Probability which is due to Kolmogorov. Then, we analyze the elementary properties of probability, namely Boole's inequality, continuity, monotonicity; the Borel-Cantelli lemmas will be stated and prooved.
A huge part of the class will be devoted to random vectors in a n-dimensional Euclidean space and their transformations, including some hints on measure theory. Moreover the conditional expectation will be introduced and analyzed in detail.
In the second part of the lectures, we focus on convergences of random variables: in distribution, in probability, almost surely and in mean. Besides, we will prove and state the limit theorems of probability and their consequences.

Finally, the general defintion of Gaussian random vectors will be provided, along with suitable applications.
Many exercises will be solved during the whole course.

PART 2. We will cover the basic principles of the Monte Carlo method, the theoretical basis of the random numbers generators as well as the fundamental concepts of resampling techniques as we discuss bootstrap and jackknife. Algorithms for iterative maximum likelihood estimation are introduced in certain examples.

Detailed program

PART 1. The first part of the course will cover the followin topics.

1. **INTRODUCTION**. Some historical hints on probability. The definitions of probability: classical, frequentist and subjective. The principle of coherence by B. de Finetti and its consequences. The axiomatic definition by Kolmogorov.
2. **AXIOMS OF PROBABILITY**. The axiomatic definition of probability and the consequences: monotonicity, continuity, Boole's inequality, etc.. The Borel-Cantelli lemmas. Conditioning and independence of events.
3. **RANDOM VECTORS AND RANDOM VARIABLES**. Definitions: random vectors (discrete and continuous case). Distributions and cumulative distribution functions. Relations between random variables: conditioning and independence. Transformations of random vectors.
4. **EXPECTED VALUES**. Expected values, variance and covariance. Markov inequality. The conditional expectation and its properties.
5. **MEASURE THEORY: HINTS**. The probability is a measure. The Lebesgue integral and the expected value. General defintion of conditional expected value given a sigma-algebra.
6. **CONVERGENCES OF RANDOM VARIABLES**. Convergences of random variables: in distribution, in probability, in mean and almost surely. Relations among convergences. The weak law of large numbers, the strong law by Kolmogorov (without proof).
7. **GENERATING FUNCTIONS**. The characteristic function and the moment generating function. The Lévy continuity theorem. The central limit theorem and the delta method.
8. **GAUSSIAN RANDOM VECTORS**. Gaussian random vectors: general defintion based on characteristic functions.

PART 2. The second part of the course will cover the following topics:

- Random numbers generation for uniform, non-uniform, discrete and continuous distributions
- Introduction to Monte Carlo simulation and Monte Carlo Integration
- Variance reduction techniques
- Resampling Techniques: bootstrap and jackknife
- Bootstrap confidence intervals
- Bootstrap Hypothesis Testing

Prerequisites

For the first part, the knowledge of the topics of Mathematical Anaysis (I and II) and Probability is required.
As for the second part, the student is required to have at least the knowledge of BSc courses on probability calculus, statistical inference, basic programming skills with R.

Teaching methods

The teaching methods of the two parts of the course are reported in the respective syllabus.

Assessment methods

PART 1. The exam is written, the oral test is not mandatory. In the written test, the student is asked to solve exercises and to answer some questions concerning probability theory. The exercises aims to ensure the ability of the students to apply the concepts of probability, whereas the theoretical questions aim to verify the knowledge of the notions of Probability. The theoretical questions may also focus on proofs.
The oral test is optional, and it may be requested by the student or by the instructor some days after the written test. The oral exam will focus on questions of the theory developed during the course. If the written test has been held online (due to Covid reasons), then the oral test is mandatory.
In the period of Covid emergency, the written and oral examination will be held via Webex and Esamionline.

PART 2.
Attending students: written exam and computational part with R.
Non-attending students: written exam and computational part with R.
During the exam, the correctness and clarity of the answers will be evaluated. The exam aims to assess the skills described in the learning objectives.
The written exam consists of 3 open-ended questions, including theoretical questions and exercises to be performed using R/RStudio through the Piattaforma degli Esami Informatizzat.
Students and the instructor may request an optional oral exam covering the entire program.
The use of texts or any other materials is not permitted during the exam, except for the codes provided by the instructor at the beginning of the exam.
The use of mobile phones or any digital support is not allowed during the exam.

Textbooks and Reading Materials

PART 1.
Theory:

- G. Dall'Aglio (2003). Calcolo delle Probabilità. Zanichelli, terza edizione.
- Grimmett G. and Stirzaker D. (2001). Probability and random processes. Oxford University Press.
  Exercises:
- Epifali, I. e Ladelli, L. (2021). Esercizi di probabilità per l'ingegneria, le scienze e l'economia. Edizioni La Dotta.
- Grimmett G. and Stirzaker D. (2000). One Thousand Exercises in Probability: Third Edition. Oxford Univseristy Press.

PART 2.

- Lecture notes provided by the instructor
- Robert, C.P. e Casella, G. (2009), Introducing Monte Carlo Methods with R, New York: Springer-Verlag
- Davison and Hinkley (1997). Bootstrap Methods and their Applications, Chapman and Hall.

Semester

The course is delivered in the first semester.

Teaching language

Italian

Sustainable Development Goals

QUALITY EDUCATION