UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Probabilità Applicata

2627-1-F8206B003-F8206B003-1

Learning objectives

The course in Applied Probability aims to provide students with advanced knowledge of probability theory, essential for understanding statistical methodology and applying it in the economic, business, and financial domains.

In terms of knowledge and understanding, the course introduces the fundamental notions of probability calculus, with particular emphasis on Kolmogorov's axiomatization. Topics covered include Gaussian vectors, random variables, and random vectors. Various modes of convergence for random variables and vectors will be studied, including limit theorems for sequences of random variables.

With regard to the ability to apply knowledge and understanding, students will learn how to determine the distribution of random vectors (both discrete and continuous). They will be able to study the convergence of sequences of random variables using advanced probabilistic tools such as the cumulative distribution function, the moment generating function, and the characteristic function. Finally, students will be able to work with Gaussian vectors.

As for independent judgment, students will be capable of critically evaluating results obtained through probabilistic analyses, correctly applying theorems to study convergence or the probability distribution of random vectors.

In terms of communication skills, the course will enable students to clearly and rigorously present complex concepts from probability theory, using appropriate mathematical language grounded in measure theory.

Finally, with respect to learning skills, students will be equipped to use probabilistic tools to understand: advanced methodologies in multivariate statistical analysis, advanced methods of statistical inference in both classical and Bayesian frameworks, spatial statistical models, models for high-dimensional data, and complex sampling designs.

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Contents

We start with the different defintions of Probability, and then we move to introduce the axiomatic definition of Probability which is due to Kolmogorov. Then, we analyze the elementary properties of probability, namely Boole's inequality, continuity, monotonicity; the Borel-Cantelli lemmas will be stated and prooved.
A huge part of the class will be devoted to random vectors in a n-dimensional Euclidean space and their transformations, including some hints on measure theory. Moreover the conditional expectation will be introduced and analyzed in detail.
In the second part of the lectures, we focus on convergences of random variables: in distribution, in probability, almost surely and in mean. Besides, we will prove and state the limit theorems of probability and their consequences.
Finally, the general defintion of Gaussian random vectors will be provided, along with suitable applications.
Many exercises will be solved during the whole course.

Detailed program

1. **INTRODUCTION**. Some historical hints on probability. The definitions of probability: classical, frequentist and subjective. The principle of coherence by B. de Finetti and its consequences. The axiomatic definition by Kolmogorov.
2. **AXIOMS OF PROBABILITY**. The axiomatic definition of probability and the consequences: monotonicity, continuity, Boole's inequality, etc.. The Borel-Cantelli lemmas. Conditioning and independence of events.
3. **RANDOM VECTORS AND RANDOM VARIABLES**. Definitions: random vectors (discrete and continuous case). Distributions and cumulative distribution functions. Relations between random variables: conditioning and independence. Transformations of random vectors.
4. **EXPECTED VALUES**. Expected values, variance and covariance. Markov inequality. The conditional expectation and its properties.
5. **MEASURE THEORY: HINTS**. The probability is a measure. The Lebesgue integral and the expected value. General defintion of conditional expected value given a sigma-algebra.
6. **CONVERGENCES OF RANDOM VARIABLES**. Convergences of random variables: in distribution, in probability, in mean and almost surely. Relations among convergences. The weak law of large numbers, the strong law by Kolmogorov (without proof).
7. **GENERATING FUNCTIONS**. The characteristic function and the moment generating function. The Lévy continuity theorem. The central limit theorem and the delta method.
8. **GAUSSIAN RANDOM VECTORS**. Gaussian random vectors: general defintion based on characteristic functions.

Prerequisites

Knowledge of the topics of Mathematical Anaysis (I and II) and Probability of the Bachelor in Statistics.

Teaching methods

Class lectures. The lectures will be in-person, during the lectures several exercises will be done.

Assessment methods

The exam is written, the oral test is not mandatory. In the written test, the student is asked to solve exercises and to answer some questions concerning probability theory. The exercises aims to ensure the ability of the students to apply the concepts of probability, whereas the theoretical questions aim to verify the knowledge of the notions of Probability. The theoretical questions may also focus on proofs.

The oral test is optional, and it may be requested by the student or by the instructor some days after the written test. The oral exam will focus on questions of the theory developed during the course.

Textbooks and Reading Materials

Theory:

- G. Dall'Aglio (2003). Calcolo delle Probabilità. Zanichelli, terza edizione.
- Grimmett G. and Stirzaker D. (2001). Probability and random processes. Oxford University Press.

Exercises:

- Epifali, I. e Ladelli, L. (2021). Esercizi di probabilità per l'ingegneria, le scienze e l'economia. Edizioni La Dotta.
- Grimmett G. and Stirzaker D. (2000). One Thousand Exercises in Probability: Third Edition. Oxford Univseristy Press.

Semester

Fall semester.

Teaching language

Italian.

Sustainable Development Goals

QUALITY EDUCATION