UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Matematica per l'Economia M

2627-1-F8206B025

Learning objectives

The course in Mathematics for Economics M aims to provide students with the fundamental skills for mathematical analysis applied to economic models, developing the ability to understand and apply mathematical techniques in economic contexts. In particular, the educational objectives include the acquisition of knowledge on differential equations and dynamic systems, with particular attention to explicit resolution, qualitative analysis and stability of solutions, as well as the understanding of existence and uniqueness theorems. The course also aims to develop skills in the field of dynamic optimization and optimal control, fundamental for the analysis of complex economic models, through the application of the Pontryagin maximum principle and optimality conditions. Finally, it intends to introduce students to the concepts of measure and integration theory, with particular attention to the Lebesgue measure and its properties, to foster a deep understanding of the advanced mathematical tools used in economics, quantitative finance, and social sciences.

These skills are closely linked to the "Statistics" learning area of the Master's Degree in Statistical and Economic Sciences, as they provide the mathematical and analytical foundations necessary for advanced statistical analysis, data modeling and interpretation of complex phenomena, promoting an integrated approach between mathematical theory and statistical applications.

Students will acquire theoretical and practical skills, developing the ability to apply this knowledge to real problems, critically interpret results and propose methodologically sound solutions. The training path promotes independent judgment in the use of mathematical and statistical tools, allowing students to become more confident and independent professionals. The course also contributes to consolidating learning and updating skills in the field of advanced methods of mathematical and statistical analysis, in line with the objective of lifelong learning.

Contents

The contents consist of three parts. The first and the second one are strictly intertwined, whereas the third one, besides connections with the second part,

provides useful notions for such courses as Finanacial Mathematics M.

In the First Part, basic elements of the theory of ordinary differential equation systems are provided.

In the Second Part, an approach to (continuous time) optimal control problems is presented, along with a solution existence result.

In the Third Part, basic elements of measure theory and of integration theory are provided. As a special case, the Lebesgue integral is considered, with special emphasis to convergence theorems (monotone and dominated).

Detailed program

Part I (ODE):

- Differential equations in mathematical economics, Cauchy problems and related solution notion.
- Reduction to first order ODE of higher order ODE.
- Solving explicitly classes of differential equations: separable equations, linear equations, Bernoulli's equations, homogeneous equations, exact equations.
- Application to specific models (market price dynamics, Solow model of economic growth).
- Global and local solution existence and uniqueness for a Cauchy problem.
- Equilibria and their stability (in the Lyapunov sense, local and global asymptotic).
- Elements for a qualitative analysis of autonomous ODE.
- Linear ODE systems: solution methods and stability.

Part II (Optimal control):

- Problem statement.
- The Pontryagin maximum principle (the linear dynamics case and beyond).
- Sufficient optimality conditions (Mangasarian condition and Arrow condition).
- Applications to economical models (optimal selling strategies, selling maximization).
- The simplest problem of the calculus of variations as a special optimal control problem and its application (an optimal consumption/investment model).
- Existence of an optimal control (Filippov's theorem).

Part III (Selected topics in measure theory):

- Algebra and ?-algebra, generated ?-algebra.
- Measures and their properties.
- The Lebesgue measure on **R**n.
- Measurable functions and their properties.
- Integral over a measure space and its properties.
- Integral functions and absolutely continuous functions.
- Convergence theorems (Lebesgue's dominated convergence theorem and B. Levi's monotone convergence theorem).
- Riemann vs Lebesgue integral.

Prerequisites

No official prerequisite. Nevertheless, a refreshement concerning the following topics in Mathematics, typically

learnt in basic courses of calculus for undergraduate students, is strongly advised:

- Basic notions about complex numbers;
- Integration of functions of one real variable;
- Multivariable calculus;
- Matrix calculus with basic elements of linear algebra;
- Eigenvalues and matrix diagonalization methods;
- Quadratic forms;
- Convexity/concavity for sets and functions.

Teaching methods

All lessons are held in person in the following delivery mode:
10 lessons of 2 hours and 6 lessons of 2 hours, all delivered in person.

During the teaching period, some exercises will be proposed to be autonomously solved by students, in preparation of the exam. Some of them will be then discussed in special sessions by the teacher.

Assessment methods

The exam will be in written form and, in case of passing the written test with a sufficient grade (>=18/30), in oral form upon request of the student or the teacher. There are no partial tests in progress.

The format of a written test essentially includes the following types of questions:

- the resolution of 3 exercises/problems;
- a detailed discussion of one of the models presented in the course;
- the detailed exposition of some arguments of the theory and their application in specific cases (open questions).

In carrying out an exam, the ability to analyze and classify a proposed problem, the ability to choose and apply the resolution methodologies proposed in the theory, the depth, precision and completeness of the exposition in the discussion of models and of the theoretical apparatus developed during the course will be evaluated.

Specific material for exam simulations will be provided by the teacher.

The final assessment is graded on a 30-point scale.

Both the written examination and oral examination (if any) are assessed according to the following grading criteria, based on these parameters:

Conceptual knowledge and understanding
Ability to apply knowledge and understanding
Communication and argumentation skills
Learning, self-assessment, and self-regulation skills

Grade **< 18**

Knowledge and Understanding:

The student identifies only some of the characteristics of the concepts. Connections between concepts are fragmented and only weakly supported by theoretical knowledge.

Ability to Apply Knowledge and Understanding:
The student identifies only some relevant elements in the analysis of a problem or exercise, without being able to integrate them into a coherent analysis.

Communication and Argumentation Skills:
In the written and oral examination, the student develops a basic argument lacking logical structure and characterized by numerous inaccuracies in presentation.

Learning, Self-Assessment, and Self-Regulation Skills:
The student is able to reconstruct only some aspects of their learning and professional development process.

Grade **18–22**

Knowledge and Understanding:
The student recognizes and describes most conceptual characteristics and is able to provide a relatively coherent explanation, although with some inaccuracies. Theoretical references are present but not always used rigorously.

Ability to Apply Knowledge and Understanding:
The student is able to identify a significant number of relevant elements and provide a partial explanation, while still showing some gaps in the analysis.

Communication and Argumentation Skills:
In the oral examination, the student develops a basic argument with a minimal structure, although some inaccuracies remain.

Learning, Self-Assessment, and Self-Regulation Skills:
The student demonstrates a basic awareness of their learning process and is able to identify essential connections among learning experiences, although with some inaccuracies.

Grade **23–27**

Knowledge and Understanding:
The student demonstrates a thorough understanding of the conceptual characteristics. In the written examination or oral discussion, explanations are well articulated and supported by an appropriate use of theoretical references.

Ability to Apply Knowledge and Understanding:
The student accurately identifies the essential elements of a mathematical problem. Knowledge is applied with methodological rigor, although not always consistently.

Communication and Argumentation Skills:
In the written examination, the student develops a coherent and well-organized argument, demonstrating good command of language and a solid logical structure. Communication is clear and effective.

Learning, Self-Assessment, and Self-Regulation Skills:
The student analyzes their learning process in a clear and structured manner, highlighting meaningful relationships among different stages of development and demonstrating good critical reflection skills.

Grade **28–30**

Knowledge and Understanding:
The student demonstrates complete mastery of the concepts, establishing complex connections and providing comprehensive explanations. Theoretical references are used appropriately and rigorously.

Ability to Apply Knowledge and Understanding:
The student demonstrates advanced problem-analysis skills, identifying and interpreting all key elements thoroughly. Knowledge is applied with methodological rigor, supported by a solid and well-developed argument.

Communication and Argumentation Skills:
In the written or oral examination, the student develops a strong and sophisticated argument, characterized by rigorous logical organization and a high degree of textual coherence. The presentation is fluent and well structured.

Learning, Self-Assessment, and Self-Regulation Skills:
The student demonstrates advanced self-reflection skills, providing a detailed and in-depth analysis of their learning and professional development process. Connections between learning experiences and theoretical concepts are clear, coherent, and rigorous.

Textbooks and Reading Materials

Lecture notes and exercises are provided during the course.

Some further reading:

1. **A. Guerraggio - S. Salsa, *Metodi matematici per l'economia e le scienze sociali*, G. Giappichelli Editore, Torino, 1997.**
2. **K. Sydsæter - P. Hammond - A. Seierstad - A. Strøm, *Further Mathematics for Economic Analysis*, Prentice Hall, Harlow, 2008.**

Semester

The course is scheduled in the second half of the second semester.

Teaching language

Italian.

Sustainable Development Goals

QUALITY EDUCATION | RESPONSIBLE CONSUMPTION AND PRODUCTION