UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Metodi Numerici Avanzati per Equazioni alle Derivate Parziali

2627-1-F4002Q020

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims at providing knowledge on some important advanced aspects of the finite element method, building a strong theoretical basis but also a good critical sense for applications. It will also build the skills needed to understand, analyse and compare the different methods, in addition to implementing and using them in the computer.

The course will be in italian, but it may shift to english if there are foreign students.

Learning Objectives according to the Dublin Descriptors.

1. Knowledge and understanding
   Students will gain an advanced understanding of numerical methods for solving partial differential equations (PDEs), with a focus on the finite element method. The course covers both theoretical aspects—such as stability and convergence—and modeling issues related to time-dependent (parabolic) problems and mixed formulations, which are essential in many scientific and engineering applications.

2. Applying knowledge and understanding
   Students will be able to effectively apply the studied numerical methods to solve complex real-world problems, implementing them in computing environments (such as MATLAB). They will also critically compare different numerical approaches.

3. Making judgements
   The course encourages the development of independent judgement in analyzing and evaluating numerical methods for PDEs. Students will be able to critically assess the strengths and limitations of the methods and choose the most appropriate strategies.

4. Communication skills
   Students will learn to communicate theoretical and computational results clearly and effectively, both orally and in written form. Emphasis will be placed on using precise mathematical and computational terminology,

even in interdisciplinary contexts.

5. Learning skills
   By the end of the course, students will have acquired the ability to continue learning independently, including exploring new numerical methods for PDEs, and engaging with advanced research topics and potential applications.

Contents

This course is about the approximation of problems in partial differential equations through the finite element method, and can be considered a second and more advanced stage of the course "Approximation of Partial Differential Equations". In particular, the course will treat important topics such as time dependent problems and problems in mixed form, that play a key role in many applications (such as fluidodynamics, diffusion problems in porius media, electromagnetism). Part of the course will be in the computer lab (MATLAB).

Detailed program

Brief review of the fundamental notions of the finite element method and main results for standard elliptic problems. The (non-stationary) heat diffusion problem, discretization in time and space, theoretical analysis of the method, implementation in MATLAB. A posteriori error analysis in the stationary case, theoretical analysis, implementation, adaptive algorithm. Problems in mixed form, Stokes as a model problem, discretization and difficulties, general theory of mixed methods, implementation. Diffusion in mixed form, theoretical analysis, implementation. The Navier-Stokes problem and its discretization with finite elements. Possible additional topics, such as problems in the field of electromagnetism, may be treated at the end of the course.

Prerequisites

Basic notions of functional analysis are needed. It is moreover required to have followed the course "Numerical Methods for Partial Differential Equations". The course will have a strong theoretical component.

Teaching form

Standard blackboard lessons and computer practice labs.

Textbook and teaching resource

- D. Braess, “Finite Elements: theory, fast solvers, and applications in solid mechanics”, Cambridge University Press (alternative: P.Ciarlet “The finite element method for elliptic problems” oppure S.Brenner e R.Scott, “The mathematical theory of finite element methods”)

- D. Boffi, F. Brezzi, M. Fortin, “Mixed finite element methods and applications”, Springer
- V. Thomee, “Galerkin Finite Element Methods for Parabolic Problems”, Springer

Semester

First semester.

Assessment method

The exam is an oral examination, and is divided into two parts. In the first part, the student presents a matlab laboratory project, that the student choses among some projects proposed by the teacher at the end of the course. The students can work in groups of 1-3 members for the development of the project (is thus allowed to work individually or as a team, but the discussion will be anyway personal). The second part of the examination is an evaluation of the critical and operational knowledge of the definitions, results and proofs presented during the course. There relative weight of the two parts, project and theory, is roughly 40% and 60%, respectively.

There will not be any mid-course evaluation/exam during the course.

Office hours

Email appointment.

Sustainable Development Goals