UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Topologia Algebrica e Computazionale

2627-1-F4002Q034

Aims

The aim of the course is to take some classical topics in algebraic and computational topology of simplicial complexes, introducing homology theory, cohomology theory, with some recent applications.

Learning Outcomes (according to the Dublin Descriptors)

1. Knowledge and understanding
   The student will acquire a solid and theoretically grounded understanding of classical and computational algebraic topology, focusing on homology and cohomology of simplicial complexes, together with insight into their recent applications in mathematics and interdisciplinary fields.

2. Applying knowledge and understanding
   The student will be able to autonomously apply homological and cohomological methods to the study of discrete topological spaces and computational models, addressing both theoretical aspects and effective algorithms, including the use of mathematical software tools.

3. Making judgements
   The student will develop the ability to critically analyse topological problems and combinatorial structures, evaluate the effectiveness of computational techniques and theoretical models used, and compare alternative approaches to problem solving.

4. Communication skills
   The student will be able to present concepts and results in algebraic and computational topology with clarity and rigour, in both formal academic contexts and seminar discussions, using appropriate abstract and computational mathematical language.

5. Learning skills
   The student will develop the ability to independently explore further theoretical and applied developments in algebraic and computational topology, in view of future research or interdisciplinary applications.

Contents

Simplicial complexes, simplicial homology of polyhedra, triangulable manifolds, singular homology, and the Mayer–Vietoris and Excision theorems.
Applications to data analysis and dynamical systems.

Detailed program

Review of category theory and functors. Review of module theory over a PID and the structure theorem for finitely generated modules. Homological algebra: exact sequences and chain complexes; homology, chain homotopies, and reduced homology.
Simplicial complexes, geometric realization, and absolute and relative simplicial homology; barycentric subdivision, simplicial approximation, and functoriality; the Euler–Poincaré characteristic. Singular homology, homotopy invariance, and relative homology. The Mayer–Vietoris and Excision theorems, with applications to the computation of the homology of spheres and surfaces. Cell attachments and CW complexes. Homology of real and complex projective spaces. Comparison between simplicial and singular homology, and the Eilenberg–Steenrod axioms.

Computational homology: normal forms and computing algorithms. Homology with coefficients. Products of polyhedra and homology. Homology of maps. Cohomology ring, cap product. Triangulable manifolds. Surfaces and classification. Poincaré Duality. Fundamental group of polyhedra. Fundamental group and homology. Applications to: computational homology, persistent homology, data analysis and dynamical systems.

Prerequisites

Basic topics covered in bachelor courses of geometry, analysis and algebra

Teaching form

A hybrid teaching approach is used, that combines lecture-based teaching (DE) and interactive teaching (DI). DE involves detailed presentation and explanation of theoretical content. DI includes active student participation through exercises and problems, short presentations, group discussions, and group or individual work. It is not possible to precisely determine in advance the number of hours dedicated to DE and DI, as these methods are dynamically intertwined to adapt to the course's needs and promote a participatory and integrated learning environment, combining theory and practice.

Lectures (56 hours) are conducted in person and are primarily in Italian, and when necessary, in English.

Textbook and teaching resource

For the first part of Algebraic Topology, the course will follow the lecture notes uploaded on the e-learning page "Appunti di Topologia Algebrica".

Additional reference texts include:

Rotman, J.J., Advanced Modern Algebra, Graduate Studies in Mathematics, American Mathematical Society, 2010.
(for the very first part covering algebraic prerequisites)

Rotman, J.J., Algebraic Topology: An Introduction, Graduate Texts in Mathematics, Springer-Verlag, 1998.
Munkres, J.R., Elements of Algebraic Topology, Addison-Wesley Pub., 1984.
(for the subsequent part on simplicial and singular homology theories)

For the second part, on Computational Topology, the course will rely on the following text:

Ferrario, D.L., Piccinini, R.A., Simplicial Structures in Topology, CMS Books in Mathematics, Springer, New York, 2011. xvi + 243 pp. ISBN: 978-1-4419-7235-4.

Plus, some notes shared on the e-learning page.

Semester

2S

Assessment method

Oral examination on the topics covered in the course, with in-depth analyis and re-elaboration of them with a personal perspective. The date and the content of the seminar, which is part of the exam, have to be first discussed with the teacher.

Office hours

By appointment.

Sustainable Development Goals

GOOD HEALTH AND WELL-BEING | QUALITY EDUCATION | INDUSTRY, INNOVATION AND INFRASTRUCTURE