UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Geometria Algebrica e Complessa

2627-1-F4002Q009

Aims

The course aims to introduce students to the fundamental concepts and methods of classical Algebraic Geometry, providing the necessary tools to understand the deep connection between the geometric properties of varieties and the algebraic properties of their associated rings.
In further detail, the learning outcomes are:
Knowledge and understanding: By the end of the course, students will have acquired: the structure of affine and projective varieties; the algebra-geometry correspondence based on Hilbert's Nullstellensatz; the concept of birational transformation; and the notion of dimension for an algebraic variety.
Applying knowledge and understanding: Students will be able to translate geometric problems into the language of ideals and vice versa; perform studies on the local properties of a variety; and analyze singularities.
Making judgements (Autonomy of judgement): The course aims to develop the ability to critically analyze and form independent judgements based on the tools learned.
Communication skills: The acquired tools will enable students to formulate clear and rigorous arguments to describe, both orally and in writing, the properties of algebraic varieties.
Learning skills: The course will provide the theoretical and methodological tools necessary to independently pursue the study of advanced topics in Algebraic Geometry.

Contents

- Plane algebraic curves.
- Affine varieties.
- Projective varieties.
- Quasi-projective varieties.
- Singularities and dimension.
- Structure of birational maps.

Detailed program

- Plane algebraic curves: affine and projective curves, rational curves, singular points, classification of projective conics.
- Affine varieties: closed subsets of an affine space, Zariski topology, irreducibility, Nullstellensatz (Hilbert's Nullstellensatz), polynomial functions and coordinate rings, polynomial maps, affine varieties, isomorphisms between affine varieties, rational functions and maps.
- Projective varieties: closed subsets of a projective space, Zariski topology, projective version of the Nullstellensatz, projective varieties, rational and regular functions, rational maps and morphisms, birational maps, rational varieties.
- Quasi-projective varieties: product of varieties, finite maps, Noether Normalization Theorem.
- Singularities and dimension: local ring at a point, tangent space, singular points, dimension, non-singular (smooth) varieties.
- Structure of birational maps: blowups and exceptional subvarieties, normal varieties and normalization.
- Complex varieties and introduction to the GAGA principle: Algebraic varieties over the complex field and the Euclidean topology. Comparison between the Zariski topology and the classical topology. Brief introduction to analytic varieties, Chow's Theorem, and the correspondence between projective algebraic varieties and compact complex manifolds.

Prerequisites

Basic notions of algebra and linear algebra, general topology and differential calculus in several variables.

Teaching form

The course consists of in-person lectures (56 hours, equivalent to 8 CFU/ECTS).
Teaching is delivered primarily through traditional lectures, during which relevant definitions, results, and theorems are presented, along with examples and problem analyses applying the introduced concepts.
At the end of each topic covered in class, exercise sheets will be assigned and published on the course's e-learning platform. Students are expected to solve these exercises independently; they can be discussed in class upon request or during office hours.
The course is scheduled to be taught in Italian; however, it may be held in English if international students are enrolled.

Textbook and teaching resource

Reference books:

- Miles Reid, "Undergraduate Algebraic Geometry",
  London Mathematical Society Student texts 12

- Igor R. Shafarievich, "Basic Algebraic Geometry 1": Varieties in Projective Space",
  Springer

- Joe Harris, "Algebraic Geometry. A first course",
  Graduate Texts in Matematics. Springer

Semester

First semester.

Assessment method

The assessment method consists in oral exam.
There are no partial texts.
The exam consists of two parts:
-in the first one there are theorethical questions involving definitions, statements of theorems, proofs,
-in the second one there are computational questions as construction of examples and counterexamples and exercises (similar to those proposed at lectureres).
The two parts will contribute equally to the the final grade (up to 30/30). In the first part, the evaluation will take into account the knowledge and the understanding of conceptual framework of the course, as well the ability to expose it in well-organized manner. In the second part, the evaluation will take into account the ability to perform exercises, the exactness of the answers and the mathematical language used.
In order to successfully complete the exam the students need to obtained a grade of at least 18/30.
There will be 6 exam sessions (january, february, april, june, july and september,).

Office hours

Upon appointment.

Sustainable Development Goals

QUALITY EDUCATION