UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Geometry and Physics

2627-1-F4002Q010

Aims

The course has the aim of presenting the mathematical tools and the conceptual ideas that are required to understand the formulation of Einstein's gravitational field. In the second part of the course the same mathematical tools will be used to discuss some geometric aspects of the modern theory of integrable systems.

The expected learning outcomes include:

- the mastering of tensor calculus.
- the knowledge of the basic concepts of the general relativity.
- the knowledge of the basic concepts of the theory of integrable systems of hydrodynamic type.

The course aims to provide students with sufficient autonomy to be able to understand advanced scientific texts on general relativity and the modern theory of integrable systems. At the end of the course students will be able to prepare a seminar on some more advanced topics not covered in the course.

Contents

Recalls of special relativity.

Tensorial calculus. Metrics, connections and curvature.

Elements of general relativity: the equivalence principle. Curved space-time. Einstein's equations.

Systems of hydrodynamic type. Differential geometric Poisson brackets. Pencils of flat metrics, bihamiltonian structures and WDVV equations.

Detailed program

- Riemannian and pseudo-riemannian metrics. Minkowski space-time. Lorentz transformations.
- Recalls of the theory of surfaces. The first and the second fundamental forms. Gauss's egregium theorem. Gauss Peterson Mainardi Codazzi equations. Riemannian and pseudo-riemannian manifolds.
- Tensor fields : algebric theory. Algebraic operations on tensors. Lie derivative. Tensors in riemannian and pseudo-riemannian manifolds. Raising and lowering indices. Covariant derivative. Levi-Civita connection. Parallel transport and curvature. Geodesics and geodesic deviation.
- Elements of general relativity: the equivalence principle. Curved space-time. Einstein's equations.
- Systems of hydrodynamic type. Riemann invariants. Integrability conditions and generalized hodograph method.
- Differential geometric Poisson bracket. Bihamiltonian structures and flat pencils of metrics. An important exmple: the orbit space of a Coxeter group and the polynomial solutions of WDVV equations.

Prerequisites

The basic notions of Mathematical Analysis I and II, Linear algebra and Geometry, Physics I and II and Dynamical Systems and Classical Mechanics of Batchelor Degree are needed. The prior knowledge of the contents of the courses Mathematical Physics (for the second part of the course) and Geometry III (for the first part of the course) might be useful but it is not required.

Teaching form

56 hours of in-person, lecture-based teaching (8 ECTS)

The course is scheduled in Italian but could be held in English in the presence of foreign students.

Textbook and teaching resource

Selected chapters from:

- B.A. Dubrovin, A.T. Fomenko, S.P. Novikov, "Modern Geometry - Methods and Applications. Part I. The Geometry of Surfaces, Transformation Groups, and Fields", Springer Graduate Texts in Mathematics
- R. d'Inverno, "Introducing Einstein's Relativity", Oxford University Press.
- N.M.J. Woodhouse, "General Relativity", Springer Undergraduate Mathematics Series.

Semester

First semester

Assessment method

Oral examination. During the oral exam the students will be asked to discuss some topics of the program and to illustrate their meaning with significant examples. The oral exam will evaluate the knowledge of the theoretical aspects of the course, as well as the ability to expose it in a well-organized and consistent way. One of the topics of the last part of the course (general relativity or integrable systems) can be replaced by a written report and a seminar on a subject in agreement with the lecturer among a list of topics of general relativity and integrable systems. The duration of the oral exam is approximately 90 minutes including the seminar. There will be no mid-term exams.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION