UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Higher Analysis

2627-1-F4002Q014

Aims

Learning objectives according to the Dublin Descriptors:

Knowledge and understanding
Students will gain a solid theoretical understanding of the main concepts in the theory of functions of bounded variation and absolutely continuous functions in one dimension, distributions (in the sense of Schwartz), and Sobolev spaces. This knowledge includes key examples and meaningful applications across various areas of mathematics.

Applying knowledge and understanding
Students will be able to apply the acquired theoretical tools to the modelling and solution of concrete mathematical problems, including those arising in numerical analysis, mathematical physics, and probability theory. They will be capable of constructing rigorous proofs and meaningful examples, even in interdisciplinary contexts.

Making judgements
Students will develop critical thinking skills necessary to evaluate the role and limitations of modern analysis techniques when applied to advanced mathematical problems. They will be able to assess the applicability of such tools in diverse theoretical and applied settings.

Communication skills
Students will be able to clearly and accurately communicate advanced mathematical content, both in written and oral form. They will be able to present theoretical arguments and discuss applications with precision, even with audiences from related scientific fields.

Learning skills
The course will enhance students' ability to engage in independent study and lifelong learning. It will provide conceptual and technical tools essential for pursuing individual research, preparing the master's thesis, or continuing studies in doctoral programs or research careers.

Contents

Functions of bounded variation. Absolutely continuous functions. Elements of distribution theory. Sobolev spaces. Second-order elliptic problems.

Detailed program

Functions of Bounded Variation. Absolutely Continuous Functions.
Functions of bounded variation and their characterization. Vitali's covering theorem. Almost everywhere differentiability of monotone functions. Absolutely continuous functions. Characterization of absolutely continuous functions.

Elements of Distribution Theory.
Definitions and examples. Derivatives of a distribution. Convergence of distributions. Operations with distributions.

Sobolev Spaces.
Motivations, definitions, and properties. Morrey's theorem. Sobolev inequality. Sobolev embeddings. Extension operators. Trace operators.

Second-Order Elliptic Problems.
Lax-Milgram lemma. Second-order elliptic problems: variational formulation, existence of solutions. Poincaré inequality. Poincaré–Wirtinger inequality. Elliptic problems with Neumann boundary conditions.

Prerequisites

Calculus in several variables, linear algebra, fundamentals of Hilbert and L? spaces.

Teaching form

56 hours of lectures delivered in a traditional, in-person format (8 ECTS credits).

Course taught in English.

Textbook and teaching resource

- Notes available on the e-learning page of the course.
- A. Bressan. *Lecture Notes on Functional Analysis*. American Mathematical Society, 2013.
- H. Brezis. *Functional analysis, Sobolev spaces and partial differential equations*. Springer Science & Business Media, 2010.
- L.C. Evans. *Partial differential equations*, American Mathematical Society, 1998.
- G. Leoni. *A First Course in Sobolev Spaces. Second Edition*, American Mathematical Society, 2017.

Semester

I semester.

Assessment method

The exam consists of a written test and an oral discussion.

The written test verifies the level of knowledge, the ability to apply it to the resolution of exercises, the student's independence in making judgements, as well as his/her communication skills. The test is divided into two parts: the first part contains theoretical questions (statements, proofs, definitions, example/counterexamples illustrated during the course), while the second part contains exercises. The two parts will contribute equally to the determination of the final grade.

The oral exam will take place a few days after the written test. It will consist of a brief discussion and correction of the written paper, and will assess the candidate's mastery of the topics covered in it.

Office hours

Upon appointment.

Sustainable Development Goals