UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Elementary Mathematics

2627-1-F4002Q005

Aims

The aim of this course is to present some elementary topics and results in Number Theory, Topology, Geometry, and Combinatorics. The term “elementary” refers to the fact that these topics do not require advanced prerequisites. The presentation of the material is progressive, emphasizing how the introduction of the topics and the preliminary problems can be understood even by high school students. Subsequently, the same problems are developed towards deeper and more modern mathematical perspectives.

This progressive approach also aims to provide examples of topics that can be effectively presented and understood in a secondary-school context, while still allowing for a rigorous and more comprehensive mathematical treatment.

Learning outcomes according to the Dublin Descriptors

1. Knowledge and understanding

Students will acquire knowledge and understanding of fundamental concepts and results in number theory, topology, geometry, and combinatorics. They will understand how apparently simple problems may lead to deep mathematical results and how such topics can be introduced progressively, including at the high school level.

2. Applying knowledge and understanding

Students will be able to apply elementary but rigorous mathematical tools to problems arising in different areas of mathematics. They will develop the ability to recognize structures and connections between elementary notions and more advanced results, using these tools also in the preparation and presentation of the final seminar.

3. Making judgements

Through the analysis of problems and their development, students will acquire the ability to critically evaluate mathematical arguments and to identify appropriate methods for addressing complex problems. They will also be

able to assess the clarity, effectiveness, and depth of different mathematical approaches and presentations.

4. Communication skills

The seminar-based examination will strengthen students' ability to present mathematical ideas clearly and rigorously, both orally and in writing, using appropriate terminology and logical organization. Students will also develop the ability to communicate complex concepts in an accessible and effective way.

5. Learning skills

By working on problems that evolve from an elementary to a more advanced level, students will strengthen their independent learning abilities and their autonomy in the study of mathematics. They will develop the ability to deepen topics independently and to engage critically with mathematical literature at different levels.

Contents

Elementary topics in number theory, geometry, topology, and combinatorics. Progressive development of elementary mathematical problems towards more advanced results and methods.

Detailed program

Prime numbers: density of prime numbers, Bertrand's postulate, the Basel problem, Willans' formula, Dirichlet's theorem.
Elements of Ramsey theory and applications to combinatorics, geometry, and analysis.
Applications of topology.
Geometric results useful in the study of arithmetic problems, with particular emphasis on sums of squares.
Problems and examples in probability theory.
Elements of Pólya's enumeration theory.
Euler's formula and its applications.

Prerequisites

The prerequisites for this course are the contents of the compulsory undergraduate courses in mathematics.
It should be emphasized that the term "elementary", as used in the course description, should not be interpreted as meaning "simple", but rather as referring to topics that require only limited mathematical prerequisites.

Teaching form

The course is delivered in person and consists of frontal lectures in a delivery-based teaching mode (DE).
Lectures will be recorded and the recordings will be made available on the course e-learning page.

Textbook and teaching resource

Notes of the course given during the lectures.
P.Cameron, Combinatorics, topics, techniques, algorithms, Cambridge university press,
G. Travaglini, Numbers and Figures, American Mathematical Society (2023).
M. Bramanti, G. Travaglini, Studying Mathematics: The Beauty, the Toil and the Method, Springer (2018).
H. Steinhaus, One Hundred Problems in Elementary Mathematics, Dover Publications, 1967.
H. Steinhaus, One Hundred Problems in Elementary Mathematics, Dover Publications, 2011 (reprint).

Semester

First semester

Assessment method

The examination consists of an oral seminar presentation lasting 45–60 minutes on a topic chosen by the student and approved by the instructor. The topic must be consistent with the subjects covered during the course and should be developed with a progression similar to that adopted during the lectures.

The oral examination is aimed at assessing:

the understanding of the mathematical contents discussed during the course;
the ability to apply mathematical methods and tools rigorously;
the ability to organize and present a mathematical topic clearly and logically;
the ability to connect elementary aspects with more advanced developments of the theory.

The evaluation will take into account:

the mathematical correctness of the presented material;
the level of understanding and depth of analysis of the topic;
the clarity and effectiveness of the presentation;
the appropriate use of mathematical terminology.

The final grade is expressed on a 30-point scale, with a minimum passing grade of 18/30. No midterm examinations are planned.

Office hours

by appointment

Sustainable Development Goals

QUALITY EDUCATION