UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Geometric Group Theory

2627-1-F4002Q013

Aims

The goal the course is to study particular graphs called trees and the groups acting on them, with attention to the theory developed by Hyman Bass and Jean-Pierre Serre in the 1970s. This theory is fundamental in geometric group theory, which aims to recover properties of groups through their action on topological spaces. The course will also cover applications such as ends of groups, the characterization of subgroups of free groups by Stallings graphs, and the boundary of a tree.

In particular, the educational objectives described in terms of the **Dublin Descriptors**, will be:

1. **Knowledge and understanding**: The student will acquire a clear and systematic knowledge of the main concepts of Bass-Serre tree theory. They will know the language, definitions and statements of the fundamental results of Bass-Serre theory (such as graphs, trees, product constructions of groups) and understand the concepts of geometric group theory and their applications, with an eye towards continuing their studies.
2. **Applying knowledge and understanding**: The student will be able to apply the main proving techniques of the Bass-Serre theory to analyze actions of groups that act on trees and will be able to use the theoretical notions to solve advanced exercises (*problem solving*).
3. **Making judgments**: The student will learn to critically evaluate the different proving techniques and their applications and will develop a critical and autonomous approach in the study of geometric group theory.
4. **Communication skills**: The student will be able to communicate clearly and effectively the concepts and techniques of the Bass-Serre theory, using mathematical language correctly to present and discuss theoretical results and applications in an academic context.
5. **Learning skills**: The student will develop the ability to independently update themselves on the topics and developments in geometric group theory, to critically interpret scientific literature and to integrate new knowledge into their professional or research practice.

The course can be taught in English or Italian. For teaching and additional purposes, English would be preferable, but in any case the language will be discussed and decided in person during the first classes of the course.

Contents

- Basics in graph theory and trees
- Construction of Cayley graphs from groups
- Construction of free groups, free products and generalizations
- Characterization of groups acting on trees

Detailed program

- Graphs, paths, connectivity, trees
- Group actions on graphs, Cayley graphs, quotient graphs
- Free groups, diamond and ping-pong lemmas
- Free products (with amalgamation) and HNN-extensions and their realizations through groups acting on trees
- Graph of groups; the fundamental group of a graph of groups
- Characterization theorem for groups acting on trees (the fundamental theorem of Bass-Serre theory)
- Kurosh theorem for subgroups of free products
- Possible additional advanced topics at the end of the course (ends of groups, Stallings graphs, boundary of a tree, etc.)

Prerequisites

Algebra I, Geometria I.

Teaching form

56 hours of in-person, lecture-based teaching (8 ECTS)

Textbook and teaching resource

- O. Bogopolski, Introduction to Group Theory, EMS Textbooks in Mathematics, 2008.
- J. Meier, Groups, Graphs and Trees, London Mathematical Society, Student Texts, 73, CUP, 2008.
- J-P. Serre, Trees, Springer-Verlag, Berlin, 1980.
- W. Dicks, M. Dunwoody. Groups acting on graphs. Cambridge Studies in Advanced Mathematics, 17. Cambridge University Press, Cambridge, 1989
- G. Baumslag, Topics in combinatorial group theory. Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 1993

Semester

1st semester

Assessment method

The final exam will consist of a conversation in two parts:

(1) The first part is given by a 20-minute presentation about an application of or a topic related to Bass-Serre theory previously agreed with the instructor. The talk will contribute 20% to the final grade

(2) The second part is an oral discussion through questions about the content covered during class time to assess the students's acquired proficiency in explaining and applying Bass-Serre theory. The oral discussion exam will provide the remaining 80% of the final grade.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION