UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Metodi Numerici per Equazioni alle Derivate Parziali

2627-1-F4002Q021

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims to providing the knowledge of the rigorous mathematical theory of the Finite Element Method for the approximation of linear elliptic second-order partial differential equations.

At the end of the course the students will have the skills needed to understand more advanced aspects of the method, both with individual work and with other courses.

The method will be implemented in MATLAB, and with the developed codes the students will have the ability to solve simple real-life problems connected with the approximation of partial differential equations.

Contents

- Finite Difference Method
- Sobolev Spaces
- Lax-Milgram Lemma
- Galerkin methods
- Cea's Lemma
- Linear Finite Elements
- Lagrange Finite Elements of order k
- Nonconforming Finite Elements
- Finite Elements with additional regularity
- Error estimates in the energy norm
- Bramble-Hilbert Lemma
- Aubin-Nitsche duality argument for L2 error estimates
- Variational crimes and Strang's Lemmas
- the Helmholtz problem

- the SUPG stabilization
- adaptive algorithms and residual error estimators
- convergence of adaptive algorithms using Dörfler's marking strategy
- DG methods: derivation and error estimates
- iterative methods for the solution to linear system stemming from finite element discretizations

Detailed program

- **Basic concepts.** Presentation in the one-dimensional case of the techniques and the ideas which will be studied in the rest of the course.
- **Sobolev Spaces.** The natural functional environment for the mathematical analysis of the finite element method.
- **Variational Formulation of Elliptic Boundary Value Problems.** Abstract setting for the partial differential equations which will be studied in the course.
- **The Construction of a Finite Element Space.** How to build a finite element.
- **Polynomial Approximation Theory in Sobolev Spaces.** The core of the course. We will study how finite elements (in essence, continuous, piecewise smooth functions) approximate functions in Sobolev Spaces.
- **Analysis of Finite Element Methods.** Analysis of different types of finite elements (DG, nonconforming, H^1 conforming, H^2 conforming) for the approximation of solutions to various types of partial differential equations (Helmholtz problem, diffusion-convection-reaction problems, ...).
- **Residual error estimator and adaptive algorithms.** Analysis of the residual error estimator, reliability, efficienty, adaptive algorithm and their convergence.
- **Iterative methods for the solution to linear systems stemming from finite element discretizations.** Multigrid methods, conjougate gradient method, proconditioned gradient method.

Prerequisites

Courses of the Laurea Triennale. It is highly recommended the course Analisi Superiore of the 1?? semester.

Teaching form

Lessons (6 CFU), exercise classes with blackboard and computer (2 CFU).

Textbook and teaching resource

The reference books are S. C. Brenner e L. R. Scott: The Mathematical Theory of Finite Element Methods, Springer 2008; A. Ern e J-L. Guermond: Finite Elements I: Approximation and Interpolation, Springer 2021; D. Braess: Finite Elements, Cambridge University Press 2010

Teacher's notes on specific topics will also be available.

Semester

2?? semester

Assessment method

The final examination is split into two parts:

- writing and presenting a project;
- oral examination.

Mark is out of thirty. The student need to reach at least 18/30 in both parts to pass the exam.

The project consists in implementing the approximation of a problem related to partial differential equations, using the codes developed during the course. The aim is to test the ability to use the developed instruments. Group working is encouraged (max 3 students) and the quality of the exposition will be part of the mark.

The oral examination will evaluate the knowledge of the definitions, results and rigorous proofs developed in the course; the capacity to understand what are the key points of the theory will also be checked.

Office hours

Upon appointment.

Sustainable Development Goals

QUALITY EDUCATION