UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Functional Analysis

2627-1-F4002Q007

Aims

Consistently with the educational objectives of the master degree in Mathematics, this course aims to provide the student with knowledge concerning the fundamentals of Functional Analysis, the skills necessary to understand and analyze the main techniques and proofs related to the theory. The skills useful in applying them to tackle problems in different areas of mathematics will also be provided. Particular emphasis will be placed on problem solving.

**Learning Outcomes.*
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1. Knowledge and Understanding
The student will acquire a clear and systematic knowledge of the main concepts of functional analysis, particularly with respect to analysis on locally compact Hausdorff spaces, spaces of continuous functions and Lp, weak and weak (weak star) topologies, compactness in weak topologies, and Riesz representation theorems.

2. Ability to apply knowledge and understanding
The student will be able to apply the methods learned to solving exercises and problems, in contexts of varying difficulty, showing mastery of problem solving techniques and understanding of basic mathematical structures.

3. Autonomy of judgment
The student will develop the ability to understand and analyze definitions, statements, and proofs, recognizing the most appropriate conceptual tools for analyzing and solving proposed problems.

4. Communication Skills
The student will be able to expound the fundamental concepts of the course with clarity and rigor, using mathematical terminology appropriately.

5. Learning skills
The student will develop the skills necessary to pursue the study of analysis independently, with the ability to consult scientific texts, online resources, and appropriate teaching resources.

Contents

Locally compact Hausdorff spaces. Spaces of continuous and Lp functions. Weak and weak* (weak star) topologies. Compactness in the weak topologies. Riesz representation theorems.

Detailed program

Metric spaces, normed vector spaces, compactness of the closed ball and dimension.

Spaces of continuous functions, and compactness through the Ascoli-Arzelà Theorem.

Linear functionals and weak topology on a normed space. Sub-additive positively homogenous functionals. The Hahn-Banach theorem: general form. Convexity and hyperplane separation.

The weak* (weak star) topology. Bi-dual and the James embedding. The Banach-Alaoglu Theorem: weak* compactness of the closed ball in the dual space.

Brief overview of reflexivity and uniform convexity for Banach spaces.

Prerequisites

Elements of the theory of abstract integration, elements of L? space theory , elements of general topology. Basic knowledge of Banach spaces and Hilbert spaces. Problem-solving skills in mathematics.

Teaching form

56 hours of lectures, 8 CFU. Frontal and interactive lectures, in-person and mixed learning.

Lectures are organized to introduce the main theoretical concepts, to present the main ideas of the proofs of the theorems and to analyze explicit examples/problems. Exercises will be assigned in order to train the problem solving skills of the students, and to elaborate on some aspects of the theory. Lectures in video published on the e-learning platform along with interactive activities may be scheduled during the year.

All the activities are in English.

Textbook and teaching resource

Bibliographic references

- H. Brezis. Functional analysis, Sobolev spaces and partial differential equations. Universitext. Springer, New York, 2011.
- G.B. Folland. Real analysis. Modern techniques and their applications. A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1999.

- W. Rudin. Real and complex analysis. McGraw-Hill Book Co., New York, third edition, 1987
- T. Bühler and D. A. Salamon. Functional analysis. Volume 191 of Graduate Studies in Mathematics. AMS, Providence, RI, 2018.

Further material

On the e-learning page of the course, the following will be made available:

- Some lecture notes, or links to online resources
- Exercises and problems.

Semester

First semester.

Assessment method

The final exam is written, with the possibility of an oral part. The written exam consists in the resolution of exercises/problems, with the aim of testing the knowledge of the students on the topics of the course, as well as testing their problem-solving skills.

The oral part of the exam is not mandatory, but it can be asked for by the student or the teacher. It consists of a discussion about the written part of the exam, and the resolution of other exercises/problems. Without an oral exam, it is not possible to obtain a mark greater than or equal to 28.

During the teaching period, 3 partial exams will be assigned as homework, and they will be graded. Students who hand in those exams will have the option to skip the written part of the final exam, but the oral part of the exam is compulsory in this case.

Office hours

By appointment (to be scheduled via e-mail).

Sustainable Development Goals

QUALITY EDUCATION