UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Basic Calculus - 1

2627-1-E1808M007-T1

Learning objectives

1. Knowledge and understanding
   Students will acquire basic knowledge of fundamental mathematical tools, with a particular focus on real functions of one or two real variables. They will understand key concepts such as domain, limits, continuity, differentiability, and will be able to interpret these concepts both graphically and analytically. These skills are essential to understand mathematical models widely used in economics, finance, and business management.

2. Applying knowledge and understanding
   Students will be able to apply the acquired mathematical tools to analyze simple economic and managerial models. They will use functions to represent economic relationships and they will solve basic optimization problems relevant to business management and decision-making processes.

3. Making judgements
   By working on mathematical and economic problems, students will develop logical and critical thinking skills. They will be able to evaluate assumptions and outcomes of quantitative models independently, understanding their limitations and implications.

4. Communication skills
   Students will learn to communicate mathematical and quantitative concepts clearly and rigorously, both orally and in writing, using appropriate technical language. They will be able to present and discuss basic quantitative models applied to economics and business management.

5. Learning skills
   The course will provide students with the necessary foundations to successfully undertake further studies in quantitative subjects (economics, statistics, finance, econometrics). Students will develop the ability to independently learn new mathematical tools, laying the groundwork for continuous skills development.

Contents

Real functions of real variables and outlines of real functions of two real variables.

Detailed program

UNIT 1 - Real functions of one real variable.
Sets N, Z, Q, R. Sets bounded from above and from belowo; intervals; upper / lower extreme and maximum / minimum of a set.
Definition of function and sequence; calculation of the field of existence; definition of image, image set, reverse image, reverse image set, graph; use of the analytic expression of a function and a sequence. Use of the graph of a function; injective, surjective, bijective functions; functions bopunded from below and from above; lower / upper bound of a function; minimum / maximum, minimum / maximum point of a function; even / odd function; monotonicity of a function and a sequence. Operations with functions, composition, inversion. Simple transformations of graphs. Horizontal / vertical translations, horizontal / vertical reflections; partial horizontal / vertical reflections; rescaling. Composed transformations of graphs.

UNIT 2 - Limits:
Real extended line and neighborhoods; definition of internal, external, border, isolated, accumulation point; definition of limit of functions and sequences; right / left limit, limit by excess/ defect; reading limits from the graph. Uniqueness of the limit theorem(with dim.), sign permanence theorem (with proof), comparison theorem (with proof). Calculation of limits for functions and sequences.
Continuity. Algebra in extended R, determined forms, limits of exponential and logarithmic functions, arctangent. Indeterminate forms, techniques for solving some indeterminate forms (rational / irrational functions). Asymptotic equivalence and properties. Orders of infinity, hierarchies of infinities.
Negligible function (o-small). Remarkable limits and relative asymptotic equivalences. Indeterminate forms of exponential type and techniques of solution. Orders of infinitesimal, hierarchy of infinitesimal, o-small. Continuity (from right / left) and discontinuity. Classification of discontinuities. Recognition of discontinuities from the graph and from the analytical expression. Horizontal, vertical, oblique asymptotes. Weierstrass theorem with counterexamples, intermediate value theorem with counterexamples, zero theorem with counterexamples.

UNIT 3 - Derivates:
Incremental ratio and derivative of a function at a point; derivative function; derivatives of elementary functions; calculation of derivatives. Equation of the tangent line; continuity-derivability link, point of inflection to vertical tangent, of cusp, angular. Rule of de L'Hopital; Rolle's theorem (with proof) and counterexamples; Lagrange's theorem (with proof) and counterexamples; derivative of the inverse function. Monotony test (with dim.) And counterexamples; definition of relative extremes; stationary point; Fermat's theorem (with proof); definition of critical point; test of the first derivative for internal extremes. Study of the montonicity of a sequence. Criterion of successive derivatives; test of the first derivative for boundary extremes; definition of concave / convex function; first order test for concavity; second order test for concavity; definition of inflection point.
Taylor and McLaurin polynomials; reminder of Peano.

UNIT 4 -Complete study of a functions and functions of two variables
General scheme for the study of function. Analytic and graphical domains for real functions of two real variables; level curves; partial derivatives, gradient, stationary points, Hessian, classification of stationary points.

Prerequisites

Set theory. Powers, logarithms, exponentials and their properties.
First and second degree inequalities, rational inequalities, logarithmic and exponentials inequalities. Cartesian equations of the line, of the circumference, of the parabola, equation of straight line passing through two points. Basics of trigonometry.

Teaching methods

A hybrid teaching approach is used, combining lecture-based (DE) and interactive teaching (DI) methods. The DE includes the presentation and detailed explanation of theoretical content, typically occurring in the first part of the lesson. The DI involves active student participation through answering questions and problems posed by the instructor, short presentations, and group discussions, usually conducted in the second part of the lesson. The exact number of hours dedicated to DE and DI cannot be predetermined, as the methods intertwine dynamically to adapt to the course needs, fostering participatory and integrated learning by combining theory and practice.

Specifically:

-40 hours of lectures will be conducted in person with the hybrid teaching method described above.
-12 hours of exercises will be conducted in person in an interactive manner

Assessment methods

A mandatory written exam consisting of theoretical questions and exercises. The written test evaluates the formal correctness of the passages, the adequacy of the mathematical language adopted, the skills and knowledge acquired during the course. Once the written exam has been passed, the professor or student can request a supplementary oral exam. The oral exam focuses on the entire program of the course and can contribute both positively and negatively to the final grade. The course does not include the splitting of the exam into intermediate tests.

Textbooks and Reading Materials

Textbooks:
Torriero, A., Scovenna M., Scaglianti, L.: Manuale di matematica. Metodi e applicazioni. CEDAM
Scovenna, M., Grassi, R.: Matematica – Esercizi e temi d’esame. CEDAM.

Additional teaching material

Course notes and teaching material provided (available on the e-learning platform)
Texts with detailed solutions of previous years (available on the e-learning platform)

Semester

First semester, first year

Teaching language

Italian

Sustainable Development Goals

QUALITY EDUCATION