



UNIVERSITÀ  
DEGLI STUDI DI MILANO-BICOCCA

## COURSE SYLLABUS

### Calculus II

2627-2-E4103B009

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#### Learning objectives

The main objective of the course is to introduce the student to differential and integral calculus for functions of several real variables, with particular reference to those elements of the theory that find greater application in statistical and economic sciences.

The present course therefore contributes to consolidating knowledge and understanding in the field of mathematical analysis, in line with the Mathematics learning area of the three-year degree course in Statistical and Economic Sciences.

The student will revisit the notions of limit, continuity, differentiability, convexity/concavity, extremality and integrability, already encountered in the course of Mathematical Analysis I, comparing differences and analogies, in light of the changed and more varied vector-topological setting that characterizes the  $n$ -dimensional Euclidean space.

In particular, he will learn new concepts such as directional derivative, gradient, Jacobian matrix, Hessian matrix, level curve or surface, multipliers, change of coordinates, which will allow him to master the fundamental techniques of differential and integral calculus in several variables.

The acquisition of the above-mentioned elements will enable him to specifically address the following problems:

1. use partial derivatives (first and successive) to approximate a function of several variables (defined explicitly or implicitly) and control the error;
2. recognize the possible concavity/convexity property of a function through differential calculus tools;
3. determine unconstrained and constrained extrema (in the presence of various types of constraints) of a function of several variables;
4. calculate multiple integrals, both proper and generalized, using various computational techniques (reduction method, change of variables, symmetries).

The development of these skills, of use both in Statistics and in Economics areas, will be achieved through the guided resolution of exercises and problems of gradual complexity, aimed at forming an autonomous judgment on the choice of theoretical elements from mathematical analysis to be used, combined and adapted in the different case studies.

A parallel objective of the teaching is to accustom the student, both in the reading phase of a text and in the production phase, to the use of the formal language in use, in relation to the topics covered, in mathematical communication, with particular reference to:

1. definition of a mathematical concept/property and formulation of theorem statements;
2. description of the methodology for solving a problem (correct presentation of calculations and their justification, as well as logical-deductive reasoning).

The achievement of this last objective, together with the acquisition of the fundamental contents proposed in the teaching, will enable the student to independently access in-depth material on the topics (as indicated by the teacher).

## **Contents**

Elements of differential and integral calculus for functions of several real variables.

## **Detailed program**

### **I. Functions on several real variables**

- I.1 Topological-vector structure of  $\mathbb{R}^n$ ;
- I.2 Limits and continuity;
- I.3 Partial and directional derivatives, gradient;
- I.4 Differentiability and tangent hyperplane;
- I.5 Higher order derivatives and Hessian matrix;
- I.6 Taylor's formula;
- I.7 Convexity/concavity for multivariable functions;
- I.8 Unconstrained extrema.

### **II. Functions implicitly defined**

- II.1 Equation systems and level curve/surfaces;
- II. 2 Implicit function theorem;
- II.3 Constrained extrema and Lagrangian multiplier method;
- II.4 Invertibility for vector-valued functions.

### **III. Multiple integration**

- III.1 Step functions and their integral;
- III.2 Riemann integrability and integral for bounded functions over a n-dimensional interval;
- III.3 Multiple integral over simple domains and reduction method;
- III.4 Change of variable in multiple integration;

III.5 Polar coordinates in the plane and in the 3d space;

III.6 Multiple integral over unbounded domains.

## **Prerequisites**

The contents of the following courses: "Mathematical Analysis I" and "Linear Algebra".

## **Teaching methods**

All classes are conducted in person in a lecture-based format:

28 hours of lectures;

24 hours of practical exercises.

## **Assessment methods**

The assessment method is based on a mandatory written test and, in case of passing the written test with a sufficient grade ( $\geq 18/30$ ), on an optional oral test (upon request of the teacher or the student). If the conditions are met, interim partial examinations may be held.

The written tests are designed to ascertain the acquisition of theoretical skills, calculation techniques and use of the main tools, and the ability to solve problems similar to those discussed and commented on in the classroom during the lessons of the course. They are structured in:

4 PROBLEMS/EXERCISES;

1 OPEN QUESTION, possibly divided into several points.

The resolution of problems/exercises requires the rationalization of a mathematical question, the application of one or more principles, sometimes appropriately combined, as well as the use of the calculation tools learned, while in the open question a succinct but pertinent theoretical exposition is required (for example, the formal definition of notions, the formulation of statements and, where applicable, their justification, the comparison between notions, the production of examples and/or counterexamples relating to pre-established properties) of the topics in the program.

The oral exam, optional, is intended to verify the learning of all the theoretical elements proposed in class as well as the ability to apply them. It therefore includes a DISCUSSION INTERVIEW ON THE WRITTEN SUBJECT, followed by an INTERVIEW ON TOPICS COVERED IN CLASS.

If the written and oral tests are passed, the final grade will be determined by the average of the results of the written and oral tests.

The criteria followed by the examination board to evaluate the written tests and any oral test will take into account the ability to identify the themes that emerge when addressing a problem, the methodological rigor in solving problems, the ability to express quantitative concepts precisely and rigorously through formal language, and the completeness of treatment in the exposition of theoretical issues.

The final assessment is graded on a 30-point scale.

Both the written examination and oral examination (if any) are assessed according to the following grading criteria, based on these parameters:

Conceptual knowledge and understanding  
Ability to apply knowledge and understanding  
Communication and argumentation skills  
Learning, self-assessment, and self-regulation skills

#### Grade < 18

Knowledge and Understanding:

The student identifies only some of the characteristics of the concepts. Connections between concepts are fragmented and only weakly supported by theoretical knowledge.

Ability to Apply Knowledge and Understanding:

The student identifies only some relevant elements in the analysis of a problem or exercise, without being able to integrate them into a coherent analysis.

Communication and Argumentation Skills:

In the written and oral examination, the student develops a basic argument lacking logical structure and characterized by numerous inaccuracies in presentation.

Learning, Self-Assessment, and Self-Regulation Skills:

The student is able to reconstruct only some aspects of their learning and professional development process.

#### Grade 18–22

Knowledge and Understanding:

The student recognizes and describes most conceptual characteristics and is able to provide a relatively coherent explanation, although with some inaccuracies. Theoretical references are present but not always used rigorously.

Ability to Apply Knowledge and Understanding:

The student is able to identify a significant number of relevant elements and provide a partial explanation, while still showing some gaps in the analysis.

Communication and Argumentation Skills:

In the oral examination, the student develops a basic argument with a minimal structure, although some inaccuracies remain.

Learning, Self-Assessment, and Self-Regulation Skills:

The student demonstrates a basic awareness of their learning process and is able to identify essential connections among learning experiences, although with some inaccuracies.

#### Grade 23–27

Knowledge and Understanding:

The student demonstrates a thorough understanding of the conceptual characteristics. In the written examination or oral discussion, explanations are well articulated and supported by an appropriate use of theoretical references.

Ability to Apply Knowledge and Understanding:

The student accurately identifies the essential elements of a mathematical problem. Knowledge is applied with methodological rigor, although not always consistently.

#### Communication and Argumentation Skills:

In the written examination, the student develops a coherent and well-organized argument, demonstrating good command of language and a solid logical structure. Communication is clear and effective.

#### Learning, Self-Assessment, and Self-Regulation Skills:

The student analyzes their learning process in a clear and structured manner, highlighting meaningful relationships among different stages of development and demonstrating good critical reflection skills.

#### Grade **28–30**

#### Knowledge and Understanding:

The student demonstrates complete mastery of the concepts, establishing complex connections and providing comprehensive explanations. Theoretical references are used appropriately and rigorously.

#### Ability to Apply Knowledge and Understanding:

The student demonstrates advanced problem-analysis skills, identifying and interpreting all key elements thoroughly. Knowledge is applied with methodological rigor, supported by a solid and well-developed argument.

#### Communication and Argumentation Skills:

In the written or oral examination, the student develops a strong and sophisticated argument, characterized by rigorous logical organization and a high degree of textual coherence. The presentation is fluent and well structured.

#### Learning, Self-Assessment, and Self-Regulation Skills:

The student demonstrates advanced self-reflection skills, providing a detailed and in-depth analysis of their learning and professional development process. Connections between learning experiences and theoretical concepts are clear, coherent, and rigorous.

## **Textbooks and Reading Materials**

1. M. Bramanti, C.D. Pagani, S. Salsa, "Analisi matematica 2", Zanichelli, Bologna, 2009;
2. S. Salsa, A. Squellati, "Esercizi di Analisi matematica 2", Zanichelli, Bologna, 2011.

## **Semester**

The course is scheduled in first semester.

## **Teaching language**

Italian.

## **Sustainable Development Goals**

QUALITY EDUCATION | RESPONSIBLE CONSUMPTION AND PRODUCTION

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