UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematics for Teaching - Geometry

2627-1-F0602Q097

Aims

Mathematics for Teaching – Geometry, together with its companion course *Mathematics for Teaching – Algebra*, is primarily intended for prospective teachers. The two courses are entirely independent. Those who teach should have a clear understanding of the foundations of the subjects they teach and should know somewhat more than what is required by the curriculum. In teaching practice, it is often necessary to approach content from unexpected perspectives in order to achieve an effective didactic transposition. Consider, for example, some of the questions that a secondary-school teacher may encounter in everyday professional practice:

- The fundamental entities of geometry are the point, the line, and the plane. Why do mathematicians not specify their meaning through suitable definitions?
- Why are proofs assigned such a central role in geometry? Which proofs are genuinely important?
- How is the length of a line segment defined? And how is the length of a curve in the plane or in space defined?
- How is the area of a plane figure defined? And how is the area of a curved surface in space defined?
- How is the volume of a solid defined? And why are proofs of the formula for the volume of a pyramid so intricate?
- What is meant by an angle? Why does this notion give rise to so many difficulties?
- Why was it only more than twenty centuries after Euclid that mathematicians, particularly in the twentieth century, felt the need to develop axiomatic foundations for plane and solid geometry equivalent to the Euclidean framework, but based on different choices of primitive notions and axioms? As for the parallel postulate, was Euclid right or wrong?
- What are the advantages of an analytic approach to geometry? And what are its limitations?
- Geometric transformations: what are they, and what are they used for?

The course aims to provide the mathematical foundations required to address questions of this kind with confidence and understanding.

By the end of the course, students will have acquired the characteristic mathematical competences, including the ability to construct and follow mathematical arguments, to apply mathematical thinking to problem solving, and to

communicate effectively using mathematical language.

Intended Learning Outcomes. Upon successful completion of the course, students will be able to:

1. demonstrate a detailed knowledge of the geometry content taught in primary and secondary education;
2. apply the knowledge acquired to teaching practice, problem solving, and the understanding of new and unfamiliar topics;
3. exercise independent judgement and integrate acquired knowledge with the autonomous study of new topics;
4. communicate clearly and accurately using mathematical language, both with specialist and non-specialist audiences;
5. pursue further learning independently and identify appropriate sources for deepening their knowledge and applying it, for example, to teaching practice.

Contents

1. The geometry of Euclid's Elements.
2. Measurement: lengths, areas, and volumes.
3. Geometric transformations.
4. Trigonometry.
5. Analytic geometry.

Detailed program

1. Euclid's *Elements*, Book I. Definitions, axioms, postulates, and propositions. The Pythagorean Theorem.
2. The axiomatic definition of measurement. Perimeter, area, and volume of elementary geometric figures. Parallelograms, triangles, circles, prisms, cylinders, cones, and spheres. The isoperimetric problem: why are soap bubbles spherical?
3. Trigonometry. Measurement of angles in radians. Sine, cosine, and tangent.
4. Geometric transformations. Plane isometries from a synthetic perspective. Similarities. An introduction to isometries in three-dimensional space.
5. Analytic geometry. Geographical coordinates. Polar and Cartesian coordinates. Thales' theorem and the equation of the line. The Pythagorean Theorem and the equation of the circle. The Cartesian plane as a model satisfying Euclid's postulates. Non-Euclidean geometries. Geometric loci: sets of points satisfying given properties. Parabola, hyperbola, and ellipse. An introduction to geometric transformations from an analytic perspective.

Prerequisites

Knowledge of the basic mathematics covered in primary and secondary school curricula. No prior courses are required.

Teaching form

All lectures are delivered in person and include: 90% lecture-based teaching, carried out through interactive lectures (19 sessions of 2 hours each); 10% interactive teaching, consisting of pedagogical and didactic workshops (two 2-hour sessions).

Textbook and teaching resource

Textbooks

- “Elementi” di Euclide (any commented edition)
- V.Villani, "Cominciamo dal punto: Domande, risposte e commenti per saperne di più della matematica (geometria)", Pitagora (2006)

Lecture notes will be provided.

Further reading

- R.Courant, H.Robbins “What is mathematics?”.
- D.Perrin "MATHÉMATIQUES D’ÉCOLE. Nombres, mesures et géométrie", Cassini (2005). (o edizioni successive)
- M.Cazzola "Matematica per scienze della formazione primaria", Carocci (2017).
- G.Polya "How to solve it".
- G.Polya "Mathematical discovery".

Semester

First semester.

Assessment method

The assessment consists of an oral examination with open-ended questions. The achievement of the intended learning outcomes will be assessed, in particular the knowledge of the mathematical content covered in the course and the ability to use such knowledge in constructing mathematical arguments.

There are no interim tests. The final grade is expressed on a scale of 30.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION