UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematics for Teaching - Algebra

2627-1-F0602Q096

Aims

Mathematics for Teaching – Algebra is primarily intended for prospective teachers of mathematics and science. The course aims to revisit the main topics of **school mathematics** from a rigorous perspective, with particular emphasis on mathematical proofs, highlighting their theoretical, historical, and educational aspects.

1. Knowledge and understanding

- Acquire a solid understanding of the fundamental concepts of algebra and arithmetic.
- Understand the concept of number and its historical development.
- Understand the proofs of selected classical mathematical results and their significance in the development of mathematical thought.
- Become familiar with the main conceptual and educational challenges associated with the topics covered in the course.

2. Applying knowledge and understanding

- Distinguish between definitions, theorems, proofs, examples, and counterexamples.
- Use examples and counterexamples to assess the validity of mathematical statements.
- Interpret the course material from an educational perspective, identifying the essential concepts and methods for effective mathematics teaching.

3. Making judgements

- Develop the ability to critically assess the correctness of mathematical arguments.
- Analyze different ways of presenting the same mathematical content, evaluating their effectiveness, rigor, and suitability for teaching.

4. Communication skills

- Present mathematical concepts clearly, precisely, and rigorously.
- Communicate definitions, theorems, and proofs effectively, adapting the level of exposition to the intended audience while maintaining mathematical rigor.

5. Learning skills
By the end of the course, students will be able to pursue further study of the topics covered independently and to approach additional topics in school mathematics using a rigorous methodology, while appreciating their theoretical, historical, and educational significance.

Contents

1 - Elemetary logic

2 - Integral numbers

3 - Modular aritmetic

4 - Rational numbers, real numbers

5 - Polynomials

Detailed program

I. Elementary logic
1. Truth tables
2. Proofs
3. Sets
4. Functions
5. Equivalence relations
6. Natural numbers
7. Cardinality

II. Integers
1. Computations in Z
2. Divisibility
3. Prime numbers
4. p-adic valuation
5. Greatest Common Divisor and Least Common Multiple
6. The equation ax + by = c
7. Positional mumbering

III. Modular arithmetic
1. Groups
2. Rings
3. The congruence-class ring

IV. Rational and real numbers
1. Rational numbers
2. Base b expansion of rational numbers

3. Real numbers
4. Continued fractions (time permitting)

V. Polynomials
1. The ring of polynomials
2. Arithmetic of polynomials
3. Roots
4. Multiplicity
5. Polynomial interpolation

Prerequisites

Background: Basic mathematics of the elementary and secondary schools. Prerequisites: None.

Teaching form

42 hours of classroom lectures. The course is scheduled in Italian but could be held in English in the presence of foreign students.

Textbook and teaching resource

Lecture notes for the course will be provided in pdf format.

Notes of the Prof. Colzani.

R.Courant, H.Robbins “What is mathematics? An elementary approach to ideas and methods”.

C.B.Boyer “A history of mathematics”.

G.Chrystal "Algebra: An elementary text-book".

Euclid “Elements”.

L.Euler “Elements of algebra”.

G.H.Hardy, E.M.Wright “An introduction to the theory of numbers”.

G.Polya "How to solve it".

G.Polya "Mathematics and plausible reasoning".

G.Polya "Mathematical discovery".

H.Steinhaus “Mathematical snapshots”.

J.Stillwell “Elements of Mathematics: From Euclid to Gödel”.

Semester

Second semester.

Assessment method

Oral examination after the end of the lectures. The maximal grade is thirty. The student will have to demonstrate adequate understanding of the content of the course, and be able to expose clearly the knowledge acquired during the course.
There are no partial examanitions during the lectures.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION