UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Analisi Matematica I

2627-1-E4105B002

Learning objectives

1. Knowledge and understanding
   At the end of the course, students will have acquired a solid understanding of the fundamental techniques of differential and integral calculus for real functions of a real variable. This knowledge will form the theoretical basis for the formalization and analysis of quantitative models used in statistics and economics. Special attention will be devoted to understanding the concepts of limit, continuity, derivative, integral, numerical series, and elementary functions.

2. Applying knowledge and understanding
   Students will be able to consciously use the basic tools of infinitesimal calculus to:

- solve mathematical problems involving real-variable functions;
- analyze qualitative properties of a function (such as zeros, growth, decrease, relative extrema, asymptotes, symmetries, and integrability);
- apply calculus techniques in simple modeling contexts from microeconomics and basic statistics.

3. Making judgements
   Students will develop the ability to critically interpret statements expressed in mathematical language and to independently assess the logical coherence of solutions and procedures. They will also be able to select and justify the most appropriate analytical methods for studying a function or solving an applied problem, especially in statistical contexts.

4. Communication skills
   Students will acquire the ability to communicate learned mathematical content with precision and rigor, using symbolic and formal language appropriately. They will be able to clearly present problem-solving strategies, explain their choices, and effectively communicate quantitative and qualitative results even to non-specialist audiences in multidisciplinary contexts.

5. Learning skills
   The course will provide students with conceptual and methodological tools useful for independently and critically tackling the study of subsequent topics, both in mathematics and statistics. Skills in abstraction and generalization will be promoted to support active learning and understanding of more complex quantitative models.

Contents

- Subsets of the real line
- Sequences
- Series
- Limits and continuity
- Differential calculus
- Integral calculus

Detailed program

The Real Line

- Maximum, minimum, supremum and infimum of sets and functions
- Elements of topology on the real line

Sequences

- The concept of limit
- Boundedness and convergence
- Monotonic sequences
- Comparison theorems
- Notable limits
- The hierarchy of divergent and infinitesiml sequences
- Subsequences

Series

- Series with eventually constant sign terms
- Series with non-constant sign terms
- Rearrangements

Limits and Continuity

- Functions
- Limits of functions
- Algebraic and comparison theorems for limits of functions
- The theorem on the limit of a function
- Limits at infinity and asymptotes
- Continuity and theorems on continuous functions
- Notable limits
- (Appendix) Construction of the exponential and logarithmic functions

Derivatives

- Motivations and definitions
- Relationship between continuity and differentiability
- Sharp points, vertical tangent inflection points, and cusps
- Computation of derivatives
- Derivative of composite functions
- Derivative of the inverse function
- Global properties of functions
- Theorems of Fermat, Rolle, Lagrange, and Cauchy
- Convexity and differentiability
- Taylor’s formula
- Taylor series of a function

Integrals

- Indefinite integrals
- Theorems of integration by parts and by substitution for indefinite integrals
- Integration of functions
- Definite integrals
- Properties of the definite integral
- The Fundamental Theorem of Calculus
- Improper integrals
- The integral comparison test for convergence of series

Prerequisites

No inner prerequisite. A refreshement (guided by a tutor, if any) is strongly advised, which should concern the main topics typically taught at the high school. More precisely:

1. algebra: solving algebraic equations of first and second degree, product cancellation and polynomial identity principle;

2. Cartesian geometry: lines, conics (parable, ellipse, hyperbole), exponential and logarithmic functions;

3. trigonometry on the plane: angles in radiants, fundamental trigonometrical functions (sine, cosine and tangent) and related formulae;

4. unidimensional inequalities.

Teaching methods

All class lectures are in-person lessons, for a total amount of 84 hours.

Class lectures are aimed at exposing the main ideas behind a notion formulated in mathematical terms and at enabling students to adequately formalize them. In this way, students are enabled to read statements about the contents and to apply them for solving problems of various kind. In order to implement this way of content trasmission (i.e. notion-mathematical formalization-connection with other notions-calculus techniques-employement

in applications), all during the class lectures emphasis is given to discussing examples and specific concepts in illustrative cases, as well as to employment of calculus techniques and problem solving.

During the teaching period, some exercise sessions are organized helping the autonomuous learning as well as (optional) self-assessment sessions, through various groups of exercizes to be solved and then discussed with the tutor. In that occasion, students can interact with the teacher, as to detect learning criticalities.

Assessment methods

Students are supposed to pass a written examinaton. For all those students who have passed the written examination, scoring a value >=18/30, an oral examination is upon request (by the teacher and the student). Interim assessments are also organized once per academic year, usually after the first half of the course and then at the end. For interim assessments, final marks are given as an arithmetic average of the partial scores.

The written examination, both mid-term and complete, aims at certifying the student skills about theoretical contents and calculus techniques provided in the course, as well as their capability in problem solving.
It consists of 2 CLOSED QUESTIONS and 2 OPEN QUESTIONS about the main subjects of the course, aimed at checking the proficiency of the basic elements and 4 PROBLEMS/EXERCISES .
Problems/exercises require to formalize a mathematical issue, to apply and combine principles, and to perform computations by means of given calculus tools, while open questions require to expose in detail some theoretical subject (e.g. providing formal definitions, theorem statements, and, whenever requested, proofs, as well as examples and counterexamples) within the course contents.

In the (optional) oral examination students are questioned about all theoretical contents of the course. During the oral examiniation students must be able to argue the solution approaches proposed in their written test and to discuss in full details the theoretical contents of the course.
The evaluation of the oral exam will be a number between -2 and 4 to be added to the evaluation of the written exam.

In any case, students who have passed the written examination with a mark below 20/30, as well as students who wish to obtain a final mark higher than 28/30, are required to take the oral examination.

The assessment for exams (in both cases, mid-term and complete) will take into acoount the methodological rigor in problem solving and in expressing mathematical concepts as well as the completeness and depth in exposing theoretical issues.

Material for exam simulations is also provided.

Textbooks and Reading Materials

D. Addona, B. Gariboldi e L. Lorenzi, Analisi Mathetatica 1, Esculapio 2022

D. Addona, B. Gariboldi e L. Lorenzi, Analisi Mathetatica 1 - Esercizi, Esculapio 2023

M. Bramanti, C.D. Pagani, S. Salsa, Analisi Matematica 1, Zanichelli, Bologna, 2008

S. Salsa, A. Squellati, Esercizi di Analisi matematica 1, Zanichelli, Bologna, 2011

A. Guerraggio, Matematica, Pearson, 2014.

Some additional material, in particular anthologies of exercises (with solution and comments) and exam simulations, are provided in the web-page associated to the course.

For refreshing prerequisites, reference to

M. Buscema, F. Lattanzi, L. Mazzoli, A. Veredice, M. Castellani, F. Gozzi, Precorso di Matematica, Società Editrice Esculapio, Bologna, 2022

is advised.

Semester

First Semester

Teaching language

Italian

Sustainable Development Goals

QUALITY EDUCATION