UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Linear Algebra

2627-1-E4105B001

Learning objectives

The goal of this course is to present the basic results of linear algebra that are fundamental and preparatory for other courses of the three-year degree course.

In particular, the educational objectives described in the terms of the **Dublin Descriptors**, will be:

1. Knowledge and understanding: The student will acquire a clear and systematic knowledge of the main concepts of linear algebra: representation of vector spaces, systems of generators and bases, linear applications and their relationship with matrices and linear systems, complex numbers, orthogonal projections, the role of eigenvalues and eigenvectors of a matrix
2. Applying knowledge and understanding: The student will be able to apply the methods learned to the resolution of exercises and problems, even in simple application contexts, showing mastery of the calculation techniques and understanding of the basic mathematical structures. They will be able to study linear applications through the theory of matrices and linear systems, determine the best approximation of an element of a vector space among the elements of its subspace, apply the procedure of diagonalization of matrices
3. Making judgments: The student will develop the ability to understand and critically evaluate definitions, statements and demonstrations, recognizing the most suitable conceptual tools for the analysis and resolution of the proposed problems.
4. Communication skills: The student will be able to explain the fundamental concepts of the course with clarity and rigor, using mathematical language correctly.
5. Learning skills: The student will develop the skills necessary to independently continue the study of linear algebra, with the ability to consult scientific texts and adequate teaching resources.

The course allows the student to acquire solid foundations in the use of linear algebra necessary in any work context and representing an essential basis for the continuation of the university studies.

Contents

1. Systems of linear equations
2. Matrices and their operations
3. Vector Spaces
4. Linear Maps
5. Inner product spaces
6. Complex Numbers and the fundamental Theorem of Algebra
7. Spectrum of a linear map
8. Diagonalization of matrices and of linear maps
9. Quadratic Forms and signature

Detailed program

Sum and product by scalars for vectors in R ^ n and between matrices. Multiplication between matrices. Systems of linear equations, number of solutions, Gauss resolution method, complete and incomplete matrix associated to a linear system. Homogeneous linear systems and non-homogeneous linear systems. Abstract definition of vector space. Definition of vector subspace. Intersection of vector subspaces and sum of vector subspaces. Linearly dependent and linearly independent vectors. Generator Vectors. Basic definition and then dimension of a vector space. Fundamental theorems on bases, generators vectors and linearly independent vectors. Definition of vector subspace generated by a set of vectors and explicit description in case the set of vectors is finished. Determinant of a matrix. Definition of linear application and its basic properties. Core and image of a linear function, nullity theorem + rank. Matrix associated with linear applications, examples, and basic properties of the matrix associated with linear applications. Discussion between the nucleus of a linear application and the nucleus of the matrix associated to the linear application. Discussion between the image of a linear application and the span of the columns of the matrix associated with the linear application. Discussion of the relationship between the rank of a matrix and the size of the span of the rows, and the size of the span of the columns. Fundamental change formula for a linear application (only in the case where the domain coincides with the codomain). Gauss method for the calculation of the inverse matrix. Endomorphisms. Eigenvalues and eigenvectors for endomorphisms. Spectrum of matrices and linear applications. Diagonalization of matrices and linear applications. Classification of quadratic forms. Elements on complex numbers.

Prerequisites

No prerequisites are required.

Teaching methods

42 hours of in-person, lecture-based teaching (6 ECTS). Classic frontal lessons, partly devoted to the theoretical aspects of the course, and partly to the resolution of practical exercises, which allow the student to acquire a method and a logical approach in solving problems. To practice problems at home and during exams we will make use of a online platform of exercises.

Assessment methods

The exam consists of a computerized written test plus a possible oral exam. The exam consists of exercises similar to those seen in the online platform of exercises accissible from home, and questions with more theoretical content. The test is computerized, but the answer to some types of questions / exercises can also be requested on paper. The duration of the test is 2 hours and during the use of textbooks or notes and handouts is not allowed. The use of programmable calculators is not allowed.

The oral part of the exam is **not** mandatory for everyone.
-Students whose grade for the written exam is 16 or less will have to take the exam again at a later time.
-The oral part of the exam is *mandatory* for everyone whose grade for the written exam is either 17, 18 or 19.
-The oral part of the exam is *elective* for those attaining a grade is greater or equal to 27 in the written exam: such students can choose to either accept a final grade of 27 or take the oral part of the exam to achieve an even better grade (it being understood that if the oral part of the exam is unsatisfactory it can lead to a worsening of the grade and even to a failure of the whole exam). To be clear: a student achieving a 28 in the written part of the exam can choose to register a final grade of 27 (and not take the oral part of the exam) or to take the oral part of the exam: depending on the oral part of the exam the 28 can either be lowered (even to a possible failure of the exam), confirmed or improved.

- To those whose grade for the written part of the test is at least 17: the oral part of the exam must be taken in all cases that either the instructor or the student requests it be taken (if a student wants to improve the grade of the written part of the exam). For example, a student who got 24 in the written test might want to skip the oral part, but also take it to improve the grade.
- The oral part of the exam, when mandated, needs to be taken in the same round (*appello*) of the written part of the exam. In all cases, the exam ends in the same round of a written part.

During the course of the lectures students will be allowed to practice solving problems on the computerized system from their homes and, in case they work on all the problems during the required time frame, they can gain up to 2 extra points. The bonus points will be added to the grade of the written exam, allowing them to have an improved grade for the written part of the exam.

Textbooks and Reading Materials

1. Schlesinger E., Algebra Lineare e Geometria, Zanichelli, 2017 (second edition)
2. Fioresi R.,Morigi M., Introduzione all'Algebra Lineare, Casa Editrice Ambrosiana, 2021 (second edition)
3. Notes of Lecures available on this platform.

Semester

Second semester (March-April)

Teaching language

Italian

Sustainable Development Goals

QUALITY EDUCATION