2D transforms for the odometry of a differential kinematics mobile robot

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Abstract

In this document we shortly present the roto-translations involved in the odometric pose estimate for a mobile robot. This task is performed in the planar (2D-3DoF) case, and for a differential kinematics mobile robot. Therefore, the document is just presenting a verbose description of how to obtain the well-known formulas for differential kinematics odometry. The development is here presented as an exercise in representation and composition of roto-translations with homogeneous coordinates, in 2D. We first introduce the problem, then develop the roto-translations that make up the transformation between two consecutive readings of the wheel angular positions, and finally compose this transform with the previous pose. The onboard reference frame has the x-axis aiming forward and the y-axis aiming left (looking at the robot from the above), which implies the z-axis pointing toward the observer.

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I. Introduction

Suppose to have a differential kinematic mobile robot, which is the easiest to build and likely the most used configuration of all wheeled mobile robot configurations. For a short introduction and review of mobile robot configurations, see e.g., the text of R. Siegwart [siegwart].

The robot will therefore have, for traction, a left and a right wheel. It will likely have also another contact point (e.g., a castor-wheel), but, as this is not significant for odometry, in the following we'll simply avoid mentioning it. The robot will move along a trajectory that will be locally orthogonal to the line between the contact points of the two wheels. This line will be mentioned hereafter as the *wheel-baseline* of the robot. The length of the wheel-baseline is assumed known, the radius of the wheels is assumed known.

For what concerns this exercise, the robot will move in a planar environment Although this would indeed be a questionable hypothesis, should the robot be real, in this exercise we are going to rely on it.

We assume here that the reader has a clear understanding of the reasons for having a wheel-based odometric system for mobile robots; in case refer again to the text by R. Siegwart [siegwart].

The odometric system works as follows: periodically, the angular position of the wheels are read; we suppose each wheel to sport an angular position sensor, e.g., an incremental encoder. Therefore, we have a discrete-time system, i.e., time is indexed with integers: at each time t, i.e., at the end of each period, the robot is in pose R_t, while at the previous time $(t-1)$ it was in R_{t-1}.

For each reading, we know the angular orientation of each wheel with respect to the previous reading. We transform this angular rotation of the wheel about its rotation axis into the length of the arc that have been moved by the contact point of each wheel, under the following hypotheses:

- known (and constant) radius of each wheel,
- no-slippage of the wheel contact point,
- other (questionable) hypotheses concerning the quality of the contact between the wheel and the floor, which we assume perfect, for this exercise.

The transformation of the angular motion of each wheel into the length of the arc moved by each contact point could be performed considering a potentially different value, for the radius of each wheel.

We also know the previous robot pose, so that the planar motion (roto-translation) that took place during the last period, mentioned hereafter as the *one-period* transform, can be composed with the previous robot pose, to obtain the current robot pose.

We also hypothesize that the period between two consecutive readings of the position of the wheels to be so small that the motion taking place during the interval can be approximated to a constant velocity motion along a circular trajectory.

Therefore, at each period, we have the one-period roto-translation between the current and the previous pose. This roto-translation can be reduced to a pure rotation around a suitable pole (Mozzi + Chasles theorem for planar motions), the so-called Centro di Istantanea Rotazione (CIR) or Center of Instantaneous Rotation.

Therefore, the one-period transform can be obtained by the composition of three elementary transforms, refer to the sketch in Figure 1, $T_{R_t}^{CIR_t}$, $T_{CIR_t}^{CIR_{t-1}}$, and $T_{CIR_{t-1}}^{R_{t-1}}$.

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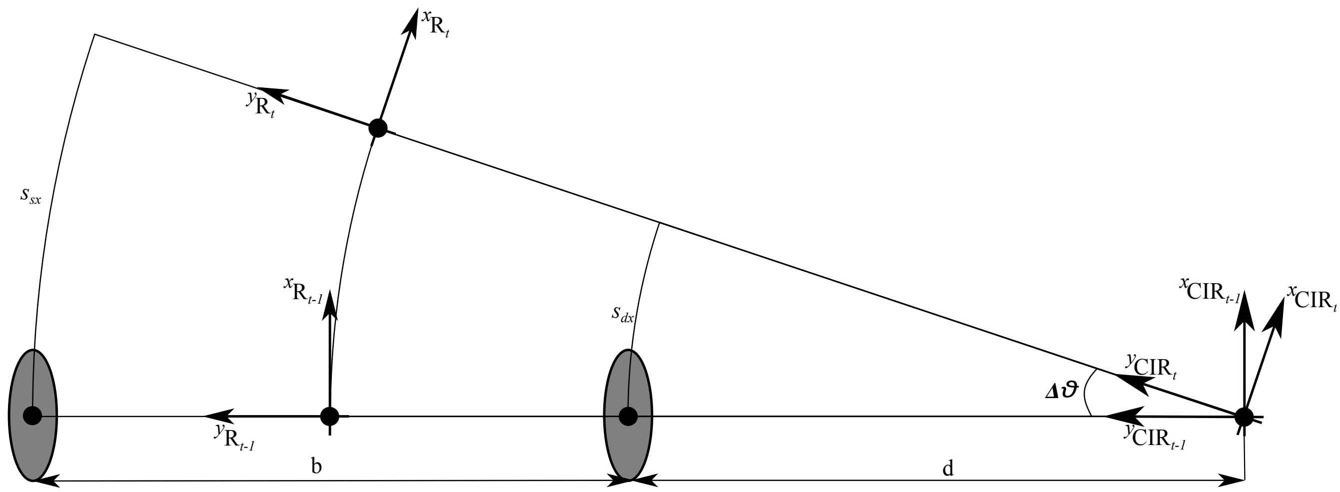

Fig. 1. Sketch of the problem; b=baseline, d=distance to CIR, s_{sx} = left arc, s_{dx} = right arc, R = robot pose.

The CIR is found at the intersection of the two wheel-baselines, one defined by the contact points of the wheels before the motion (robot pose = R_{t-1}), and the other one defined by the contact points of the wheels at the end of the motion (robot pose = R_t), see Figure 1.

The parameters of the elementary transform are d, the robot to CIR distance, and $\Delta\vartheta$, the angle of rotation about the CIR. If we know these two parameters, we can write down the elementary transforms, i.e., the components of the one=period transform.

II. Determination of the parameters of the elementary transform

To model the elementary transform $T_{R_t}^{R_{t-1}}$ we need to know the distance d of the CIR from the robot, which is along the baseline, as well as the rotation angle $\Delta\vartheta$ about the axis orthogonal to the motion plane, axis that is passing through the CIR.

We can observe that:

$$\begin{cases} s_{dx} = d\ \Delta\vartheta \\ s_{sx} = (d+b)\ \Delta\vartheta \end{cases} ; \quad \begin{cases} d = \\ \Delta\vartheta = \end{cases}$$

III. The elementary transforms

The elementary transforms required to build up the one-period transform can now be easily determined

IV. The one-period transform

We can now compose the elementary transforms to build the one-period transform

V. The odometric estimate of the robot pose

We can now pass to the last step, i.e., the composition of the one-period transform with the previous robot pose estimate, so to obtain an updated pose with respect to the world reference frame $\mathbf{T}_{R_t}^{W}$. This matrix, which is in the form of a roto-translation matrix in homogeneous coordinates in the plane, depends on the free pose parameters $[X, Y, \vartheta]^T$. Of course, from any such matrix one can extract the free pose parameters $[X, Y, \vartheta]^T$.

After giving out the odometric estimate at the current time, i.e., concluding the current time iteration, the current estimate becomes the previous estimate for the forthcoming new reading of the two arcs (s_{sx} and s_{dx}), and then the procedure is iterated.