II ESERCITAZIONE 7-11-2018

ESERCIZIO 1 Date le funzioni $f: \mathbb{R} \to \mathbb{R}$ e $g: \mathbb{R} \to \mathbb{R}$ definite da

$$f(x) = -\frac{3}{2}x + 7 \quad e \quad g(x) = x^2 - 6x$$

Scrivere l'espressione analitica delle funzioni composte

$$f[g(x)] \quad , \quad g[f(x)] \quad , \quad g[g(x)]$$

Ricordiamo che date le funzioni $f: A \subseteq X \to Y$ e $g: Y \to Z$, la funzione composta h mediante f e g $(h = g[f])$ è la funzione $h: A \subseteq X \to Z$ che ad ogni $x \in A \subseteq X$ associa $g[f(x)]$.

OSSERVAZIONE $\quad Im(f) \subseteq Dom(g)$

Consideriamo le funzioni f e g date, notiamo che Dominio e Immagine soddisfano la relazione

$$Im(f) \subseteq Dom(g) \quad ; \quad Im(g) \subseteq Dom(f)$$

$$e \quad Im(g) \subseteq Dom(g)$$

Dunque le composizioni richieste posso essere fatte.

$$(f \circ g)(x) = f[g(x)] = f[x^2 - 6x] = -\frac{3}{2}[x^2 - 6x] + 7 =$$

$$= -\frac{3}{2}x^2 + 9x + 7$$

$$(g \circ f)(x) = g[f(x)] = g\left[-\frac{3}{2}x + 7\right] = \left(-\frac{3}{2}x + 7\right)^2 - 6\left(-\frac{3}{2}x + 7\right) =$$

$$= \frac{9}{4}x^2 + \cancel{2}\left(-\frac{3}{\cancel{2}}x\right)7 + 49 + 9x - 42 =$$

$$= \frac{9}{4}x^2 - 21x + 49 + 9x - 42 = \frac{9}{4}x^2 - 12x + 7$$

$(g \circ g)(x) = g[g(x)] = g[x^2 - 6x] = (x^2-6x)^2 - 6(x^2-6x) =$

$= x^4 + (-6x)^2 + 2x^2(-6x) - 6x^2 + 36x =$

$= x^4 + 36x^2 - 12x^3 - 6x^2 + 36x =$

$= x^4 - 12x^3 + 30x^2 + 36x$

ESERCIZIO 2 Semplificare le seguenti espressioni usando i prodotti notevoli

I) $(a+b)^3 + (a-b)^3 - 6ab^2$

II) $(2x+1)^2 + (x+1)(x-1) - (x+2)(x-2)$

I) $(a+b)^3 + (a-b)^3 - 6ab^2 =$

$= a^3 + \cancel{3a^2b} + 3ab^2 + \cancel{b^3} + a^3 - \cancel{3a^2b} + 3ab^2 - \cancel{b^3} - 6ab^2 =$

$= a^3 + 3ab^2 + a^3 + 3ab^2 - 6ab^2 =$

$= 2a^3 + ab^2(3+3-6) =$

$= 2a^3 + ab^2(0) =$

$= 2a^3 + 0 =$

$= 2a^3$

II) $(2x+1)^2 + (x+1)(x-1) - (x+2)(x-2) =$

$= (2x)^2 + 2(2x)(1) + 1^2 + x^2 - 1^2 - (x^2 - 2^2) =$

$= 4x^2 + 4x \cancel{+1} + \cancel{x^2} \cancel{-1} \cancel{-x^2} + 4 =$

$= 4x^2 + 4x + 4 =$

$= 4(x^2 + x + 1)$

ESERCIZIO 3 Semplificare le seguenti espressioni usando le operazioni con frazioni algebriche

I) $\left[\left(\frac{x+6}{2}+\frac{3}{a-1}\right):\frac{x}{a-1}+3a+\frac{x(1+2a)}{2}+\frac{x(1-7a)+3}{2x}\right]\cdot\frac{2x^2}{x^2+3}$

II) $\frac{x+2}{x-3}-\frac{2-x}{1-x}+\frac{x^2+1}{x^2-4x+3}-1$

I) $\left[\left(\frac{x+6}{2}+\frac{3}{a-1}\right):\frac{x}{a-1}+3a+\frac{x(1+2a)}{2}+\frac{x(1-7a)+3}{2x}\right]\cdot\frac{2x^2}{x^2+3}=$

$=\left[\frac{(x+6)(a-1)+3\cdot 2}{2(a-1)}:\frac{x}{a-1}+3a+\frac{x(1+2a)}{2}+\frac{x(1-7a)+3}{2x}\right]\cdot\frac{2x^2}{x^2+3}=$

$=\left[\frac{ax-x+6a\cancel{-6}\cancel{+6}}{2\cancel{(a-1)}}\cdot\frac{\cancel{a-1}}{x}+3a+\frac{x(1+2a)}{2}+\frac{x(1-7a)+3}{2x}\right]\cdot\frac{2x^2}{x^2+3}=$

$=\left[\frac{ax-x+6a}{2x}+3a+\frac{x(1+2a)}{2}+\frac{x(1-7a)+3}{2x}\right]\cdot\frac{2x^2}{x^2+3}=$

$=\left[\frac{ax-x+6a+6ax+x^2(1+2a)+x(1-7a)+3}{2x}\right]\cdot\frac{2x^2}{x^2+3}=$

$=\left[\frac{\cancel{ax}\cancel{-x}+6a+\cancel{6ax}+x^2+2ax^2\cancel{+x}\cancel{-7ax}+3}{2x}\right]\cdot\frac{2x^2}{x^2+3}=$

$=\left[\frac{2ax^2+x^2+6a+3}{2x}\right]\cdot\frac{2x^2}{x^2+3}=$

$$= \frac{x^2(2a+1)+3(2a+1)}{\cancel{2}\cancel{x}} \cdot \frac{\cancel{2}x^{\cancel{2}}}{x^2+3} =$$

$$= \cancel{(x^2+3)}(2a+1) \cdot \frac{x}{\cancel{x^2+3}} =$$

$$= (2a+1)x$$

II) $$\frac{x+2}{x-3} - \frac{2-x}{1-x} + \frac{x^2+1}{x^2-4x+3} - 1 =$$

$$= \frac{x+2}{x-3} - \frac{2-x}{1-x} + \frac{x^2+1}{(x-3)(x-1)} - 1 =$$

$$= \frac{x+2}{x-3} + \frac{2-x}{x-1} + \frac{x^2+1}{(x-3)(x-1)} - 1 =$$

$$= \frac{(x+2)(x-1)+(2-x)(x-3)+x^2+1-(x-3)(x-1)}{(x-3)(x-1)} =$$

$$= \frac{\cancel{x^2}-x+2x-2+2x-6-\cancel{x^2}+3x+\cancel{x^2}+1-\cancel{x^2}+x+3x-3}{(x-3)(x-1)} =$$

$$= \frac{10x-10}{(x-3)(x-1)} =$$

$$= \frac{10\cancel{(x-1)}}{(x-3)\cancel{(x-1)}} =$$

$$= \frac{10}{x-3}$$

ESERCIZIO 4 Semplificare le seguenti espressioni con potenze e radicali

I) $\left\{\left[\left(6 \cdot 6^5 \cdot 6^8\right)^5 : \left(60^9 : 10^9\right)^7\right]^6 : \left(6^5\right)^8\right\}^9 : \left(2^5 \cdot 8^3 : 4^4\right)^3 \cdot 125^6$

II) $\left(\sqrt{x} - \sqrt{y}\right)^2 - \left(\sqrt{x} - 1\right)\left(\sqrt{x} - \sqrt{y}\right) + \left(\sqrt{y} + 2\right)\left(\sqrt{x} + 1\right) - 3\left(y + \sqrt{x}\right)$

I) $\left\{\left[\left(6 \cdot 6^5 \cdot 6^8\right)^5 : \left(60^9 : 10^9\right)^7\right]^6 : \left(6^5\right)^8\right\}^9 : \left(2^5 \cdot 8^3 : 4^4\right)^3 \cdot 125^6 =$

$= \left\{\left[\left(6^{1+5+8}\right)^5 : \left(\left(\frac{60}{10}\right)^9\right)^7\right]^6 : 6^{5\times 8}\right\}^9 : \left(2^5 \cdot \left(2^3\right)^3 : \left(2^2\right)^4\right)^3 \cdot 125^6 =$

$= \left\{\left[\left(6^{14}\right)^5 : \left(6^9\right)^7\right]^6 : 6^{40}\right\}^9 : \left(2^5 \cdot 2^{3\times 3} : 2^{2\times 4}\right)^3 \cdot 125^6 =$

$= \left\{\left[6^{14\times 5} : 6^{9\times 7}\right]^6 : 6^{40}\right\}^9 : \left(2^5 \cdot 2^9 : 2^8\right)^3 \cdot \left(5^3\right)^6 =$

$= \left\{\left[6^{70} : 6^{63}\right]^6 : 6^{40}\right\}^9 : \left(2^{5+9} : 2^8\right)^3 \cdot 5^{3\times 6} =$

$= \left\{\left[6^{70-63}\right]^6 : 6^{40}\right\}^9 : \left(2^{14} : 2^8\right)^3 \cdot 5^{18} =$

$$= \left\{ \left[6^7 \right]^6 : 6^{40} \right\}^9 : \left(2^{14-8} \right)^3 \cdot 5^{18} =$$

$$= \left\{ 6^{7 \times 6} : 6^{40} \right\}^9 : \left(2^6 \right)^3 \cdot 5^{18} =$$

$$= \left\{ 6^{42} : 6^{40} \right\}^9 : 2^{6 \times 3} \cdot 5^{18} =$$

$$= \left\{ 6^{42-40} \right\}^9 : 2^{18} \cdot 5^{18} =$$

$$= \left\{ 6^2 \right\}^9 : 2^{18} \cdot 5^{18} =$$

$$= 6^{2 \times 9} : 2^{18} \cdot 5^{18} =$$

$$= 6^{18} : 2^{18} \cdot 5^{18} =$$

$$= \left(\frac{6}{2} \right)^{18} \cdot 5^{18} =$$

$$= 3^{18} \cdot 5^{18} =$$

$$= (3 \cdot 5)^{18} =$$

$$= 15^{18}$$

II) $(\sqrt{x} - \sqrt{y})^2 - (\sqrt{x} - 1)(\sqrt{x} - \sqrt{y}) + (\sqrt{y} + 2)(\sqrt{x} + 1) - 3(y + \sqrt{x}) =$

$$= \cancel{(\sqrt{x})^2} + (\sqrt{y})^2 + 2\sqrt{x}\sqrt{y} - \left[\cancel{(\sqrt{x})^2} - \sqrt{x}\sqrt{y} - \sqrt{x} + \sqrt{y} \right] + \sqrt{y}\sqrt{x} + \sqrt{y} +$$

$$+ 2\sqrt{x} + 2 - 3y - 3\sqrt{x} =$$

$$= \cancel{x} + \underline{y} + 2\sqrt{x}\sqrt{y} - \cancel{x} + \sqrt{x}\sqrt{y} + \sqrt{x} - \cancel{\sqrt{y}} + \sqrt{y}\sqrt{x} + \cancel{\sqrt{y}} + 2\sqrt{x} +$$

$+2 - \underline{3y} - \underline{\underline{3\sqrt{x}}} =$

$= y(1-3) + \sqrt{xy}\,(-2+1+1) + \sqrt{x}\,(1+2-3) + 2 =$

$= y(-2) + \sqrt{xy}\,(0) + \sqrt{x}\,(0) + 2 =$

$= -2y + 0 + 0 + 2 =$

$= -2y + 2 =$

$= 2(1-y)$