

$$\begin{cases} IS : Y = c_0 + c_1(Y - T) + \bar{I} + d_1Y - d_2i + G \\ LM : \frac{M}{P} = f_1Y - f_2i \end{cases}$$

$$\begin{cases} Y = c_0 - c_1T + \bar{I} + G - d_2i + (c_1 + d_1)Y \\ \frac{M}{P} = f_1Y - f_2i \end{cases}$$

definiamo $c_0 - c_1T + \bar{I} + G = A$

$$\begin{cases} d_2i = A - (1 - c_1 - d_1)Y \\ \frac{M}{P} = f_1Y - f_2i \end{cases}$$

$$\begin{cases} i = \frac{A}{d_2} - \frac{(1 - c_1 - d_1)}{d_2}Y \\ \frac{M}{P} = f_1Y - f_2i \end{cases}$$

$$\begin{cases} i = \frac{A}{d_2} - \frac{(1 - c_1 - d_1)}{d_2}Y \\ \frac{M}{P} = f_1Y - f_2 \left[\frac{A}{d_2} - \frac{(1 - c_1 - d_1)}{d_2}Y \right] \end{cases}$$

Andiamo avanti con la seconda equazione:

$$\frac{M}{P} = f_1Y + f_2 \frac{(1 - c_1 - d_1)}{d_2}Y - f_2 \frac{A}{d_2}$$

$$f_1Y + f_2 \frac{(1 - c_1 - d_1)}{d_2}Y = \frac{M}{P} + f_2 \frac{A}{d_2}$$

$$Y \left[f_1 + f_2 \frac{(1 - c_1 - d_1)}{d_2} \right] = \frac{M}{P} + f_2 \frac{A}{d_2}$$

$$Y = \frac{1}{\left[f_1 + f_2 \frac{(1 - c_1 - d_1)}{d_2} \right]} \frac{M}{P} + \frac{1}{\left[f_1 + f_2 \frac{(1 - c_1 - d_1)}{d_2} \right]} f_2 \frac{A}{d_2}$$

$$Y = \frac{1}{\left[f_1 + f_2 \frac{(1 - c_1 - d_1)}{d_2} \right]} \frac{M}{P} + \frac{1}{\left[f_1 d_2 + f_2 (1 - c_1 - d_1) \right]} f_2 \frac{A}{d_2}$$

$$Y = \frac{1}{\left[f_1 + f_2 \frac{(1 - c_1 - d_1)}{d_2} \right]} \frac{M}{P} + \frac{1}{\left[f_1 d_2 + f_2 (1 - c_1 - d_1) \right]} f_2 A$$

$$Y = \frac{1}{\left[f_1 + f_2 \frac{(1 - c_1 - d_1)}{d_2} \right]} \frac{M}{P} + \frac{1}{\left[\frac{f_1 d_2}{f_2} + (1 - c_1 - d_1) \right]} A$$

Inseriamo questo valore di Y nella prima equazione del sistema sopra:

$$\begin{aligned}
i &= \frac{A}{d_2} - \left[\frac{(1-c_1-d_1)}{d_2} \left[\frac{1}{\left[f_1 + f_2 \frac{(1-c_1-d_1)}{d_2} \right]} \frac{M}{P} + \frac{1}{\left[\frac{f_1 d_2}{f_2} + (1-c_1-d_1) \right]} A \right] \right] \\
i &= \frac{A}{d_2} - \left[\frac{(1-c_1-d_1)}{d_2} \left[\frac{1}{\left[f_1 + f_2 \frac{(1-c_1-d_1)}{d_2} \right]} \frac{M}{P} + \frac{(1-c_1-d_1)}{d_2} \frac{1}{\left[\frac{f_1 d_2}{f_2} + (1-c_1-d_1) \right]} A \right] \right] \\
i &= \frac{A}{d_2} - \frac{(1-c_1-d_1)}{d_2 f_1 + d_2 f_2} \frac{(1-c_1-d_1)}{d_2} \frac{M}{P} + \left(\frac{(1-c_1-d_1)}{d_2} \frac{1}{\left[(1-c_1-d_1) + \frac{f_1 d_2}{f_2} \right]} \right) A \\
i &= - \frac{(1-c_1-d_1)}{d_2 f_1 + d_2 f_2} \frac{(1-c_1-d_1)}{d_2} \frac{M}{P} + \left(- \frac{A}{d_2} + \frac{(1-c_1-d_1)}{d_2 \left[(1-c_1-d_1) + \frac{f_1 d_2}{f_2} \right]} \right) A \\
i &= - \frac{1}{\frac{d_2 f_1}{(1-c_1-d_1)} + \frac{d_2 f_2}{d_2}} \frac{M}{P} + \left(\frac{\left[(1-c_1-d_1) + \frac{f_1 d_2}{f_2} \right] + (1-c_1-d_1)}{d_2 \left[(1-c_1-d_1) + \frac{f_1 d_2}{f_2} \right]} \right) A \\
i &= - \frac{1}{\frac{d_2 f_1}{(1-c_1-d_1)} + f_2} \frac{M}{P} + \left(\frac{\frac{f_1 d_2}{f_2}}{d_2 \left[(1-c_1-d_1) + \frac{f_1 d_2}{f_2} \right]} \right) A \\
i &= - \frac{1}{\frac{d_2 f_1}{(1-c_1-d_1)} + f_2} \frac{M}{P} + \left(\frac{1}{d_2 \left[(1-c_1-d_1) + \frac{f_1 d_2}{f_2} \right] \frac{f_2}{d_2 f_1}} \right) A \\
i &= - \frac{1}{\frac{d_2 f_1}{(1-c_1-d_1)} + f_2} \frac{M}{P} + \left(\frac{1}{(1-c_1-d_1) \frac{f_2}{f_1} + d_2} \right) A
\end{aligned}$$

$$Y^* = \frac{1}{(1-c_1-d_1) \frac{f_2}{d_2} + f_1} \cdot \frac{M}{P} + \frac{1}{(1-c_1-d_1) + d_2 \frac{f_1}{f_2}} \cdot A$$

$$i^* = - \frac{1}{f_2 + \frac{d_2 f_1}{1-c_1-d_1}} \cdot \frac{M}{P} + \frac{1}{(1-c_1-d_1) \frac{f_2}{f_1} + d_2} \cdot A$$

$$\frac{\partial Y}{\partial \frac{M}{P}} = \frac{1}{(1-c_1-d_1) \frac{f_2}{d_2} + f_1} = MOLTIPLICATORE \text{ della politica monetaria}$$

$$\frac{\partial Y}{\partial A} = \frac{1}{(1-c_1-d_1) + d_2 \frac{f_1}{f_2}} = MOLTIPLICATORE \text{ della politica fiscale}$$