

II ESERCITAZIONE

ESERCIZIO 1 Date le funzioni $f: \mathbb{R} \rightarrow \mathbb{R}$ e $g: \mathbb{R} \rightarrow \mathbb{R}$ definite da

$$f(x) = -\frac{3}{2}x + 7 \quad \text{e} \quad g(x) = x^2 - 6x$$

Scrivere l'espressione analitica delle funzioni composte

$$f[g(x)], \quad g[f(x)], \quad g[g(x)]$$

Ricordiamo che date le funzioni $f: A \subseteq X \rightarrow Y$ e $g: Y \rightarrow Z$, la funzione composta h mediante f e g ($h = g[f]$) è la funzione $h: A \subseteq X \rightarrow Z$ che ad ogni $x \in A \subseteq X$ associa $g[f(x)]$.

OSSERVAZIONE $\text{Im}(f) \subseteq \text{Dom}(g)$

Consideriamo le funzioni f e g date, notiamo che Dominio e Immagine soddisfano la relazione

$$\text{Im}(f) \subseteq \text{Dom}(g) \quad ; \quad \text{Im}(g) \subseteq \text{Dom}(f)$$

$$\text{e} \quad \text{Im}(g) \subseteq \text{Dom}(g)$$

Dunque le composizioni richieste possono essere fatte.

$$\begin{aligned} (f \circ g)(x) &= f[g(x)] = f[x^2 - 6x] = -\frac{3}{2}[x^2 - 6x] + 7 = \\ &= -\frac{3}{2}x^2 + 9x + 7 \end{aligned}$$

$$\begin{aligned} (g \circ f)(x) &= g[f(x)] = g[-\frac{3}{2}x + 7] = (-\frac{3}{2}x + 7)^2 - 6(-\frac{3}{2}x + 7) = \\ &= \frac{9}{4}x^2 + \cancel{2(-\frac{3}{2}x)7} + 49 + 9x - 42 = \\ &= \frac{9}{4}x^2 - 21x + 49 + 9x - 42 = \frac{9}{4}x^2 - 12x + 7 \end{aligned}$$

$$\begin{aligned}
 (g \circ g)(x) &= g[g(x)] = g[x^2 - 6x] = (x^2 - 6x)^2 - 6(x^2 - 6x) = \\
 &= x^4 + (-6x)^2 + 2x^2(-6x) - 6x^2 + 36x = \\
 &= x^4 + 36x^2 - 12x^3 - 6x^2 + 36x = \\
 &= x^4 - 12x^3 + 30x^2 + 36x
 \end{aligned}$$

ESERCIZIO 2 Semplificare le seguenti espressioni usando i prodotti notevoli

$$\text{I) } (a+b)^3 + (a-b)^3 - 6ab^2$$

$$\text{II) } (2x+1)^2 + (x+1)(x-1) - (x+2)(x-2)$$

$$\text{I) } (a+b)^3 + (a-b)^3 - 6ab^2 =$$

$$= a^3 + \cancel{3a^2b} + 3ab^2 + \cancel{b^3} + a^3 - \cancel{3a^2b} + 3ab^2 - \cancel{b^3} - 6ab^2 =$$

$$= a^3 + 3ab^2 + a^3 + 3ab^2 - 6ab^2 =$$

$$= 2a^3 + ab^2(3+3-6) =$$

$$= 2a^3 + ab^2(0) =$$

$$= 2a^3 + 0 =$$

$$= 2a^3$$

$$\text{II) } (2x+1)^2 + (x+1)(x-1) - (x+2)(x-2) =$$

$$= (2x)^2 + 2(2x)(1) + 1^2 + x^2 - 1^2 - (x^2 - 2^2) =$$

$$= 4x^2 + 4x + \cancel{1} + \cancel{x^2} - \cancel{1} - \cancel{x^2} + 4 =$$

$$= 4x^2 + 4x + 4 =$$

$$= 4(x^2 + x + 1)$$

ESERCIZIO 3 Semplificare le seguenti espressioni usando le operazioni con frazioni algebriche

$$\text{I)} \left[\left(\frac{x+6}{2} + \frac{3}{a-1} \right) : \frac{x}{a-1} + 3a + \frac{x(1+2a)}{2} + \frac{x(1-7a)+3}{2x} \right] \cdot \frac{2x^2}{x^2+3}$$

$$\text{II)} \frac{x+2}{x-3} - \frac{2-x}{1-x} + \frac{x^2+1}{x^2-4x+3} - 1$$

$$\text{I)} \left[\left(\frac{x+6}{2} + \frac{3}{a-1} \right) : \frac{x}{a-1} + 3a + \frac{x(1+2a)}{2} + \frac{x(1-7a)+3}{2x} \right] \cdot \frac{2x^2}{x^2+3} =$$

$$= \left[\frac{(x+6)(a-1) + 3 \cdot 2}{2(a-1)} : \frac{x}{a-1} + 3a + \frac{x(1+2a)}{2} + \frac{x(1-7a)+3}{2x} \right] \cdot \frac{2x^2}{x^2+3} =$$

$$= \left[\frac{ax - x + 6a - \cancel{6} + \cancel{6}}{2(\cancel{a-1})} \cdot \frac{\cancel{a-1}}{x} + 3a + \frac{x(1+2a)}{2} + \frac{x(1-7a)+3}{2x} \right] \cdot \frac{2x^2}{x^2+3} =$$

$$= \left[\frac{ax - x + 6a}{2x} + 3a + \frac{x(1+2a)}{2} + \frac{x(1-7a)+3}{2x} \right] \cdot \frac{2x^2}{x^2+3} =$$

$$= \left[\frac{ax - x + 6a + 6ax + x^2(1+2a) + x(1-7a) + 3}{2x} \right] \cdot \frac{2x^2}{x^2+3} =$$

$$= \left[\frac{\cancel{ax} - \cancel{x} + 6a + \cancel{6ax} + x^2 + 2ax^2 + \cancel{x} - \cancel{7ax} + 3}{2x} \right] \cdot \frac{2x^2}{x^2+3} =$$

$$= \left[\frac{2ax^2 + x^2 + 6a + 3}{2x} \right] \cdot \frac{2x^2}{x^2+3} =$$

$$= \frac{x^2(2a+1) + 3(2a+1)}{\cancel{2x}} \cdot \frac{\cancel{2x^2}}{x^2+3} =$$

$$= (\cancel{x^2+3})(2a+1) \cdot \frac{x}{\cancel{x^2+3}} =$$

$$= (2a+1)x$$

$$\text{II) } \frac{x+2}{x-3} - \frac{2-x}{1-x} + \frac{x^2+1}{x^2-4x+3} - 1 =$$

$$= \frac{x+2}{x-3} - \frac{2-x}{1-x} + \frac{x^2+1}{(x-3)(x-1)} - 1 =$$

$$= \frac{x+2}{x-3} + \frac{2-x}{x-1} + \frac{x^2+1}{(x-3)(x-1)} - 1 =$$

$$= \frac{(x+2)(x-1) + (2-x)(x-3) + x^2+1 - (x-3)(x-1)}{(x-3)(x-1)} =$$

$$= \frac{\cancel{x^2} - x + 2x - 2 + 2x - 6 - \cancel{x^2} + 3x + \cancel{x^2} + 1 - \cancel{x^2} + x + 3x - 3}{(x-3)(x-1)} =$$

$$= \frac{10x - 10}{(x-3)(x-1)} =$$

$$= \frac{10(\cancel{x-1})}{(x-3)(\cancel{x-1})} =$$

$$= \frac{10}{x-3}$$

ESERCIZIO 4 Semplificare le seguenti espressioni con potenze e radicali

$$\text{I)} \left\{ \left[(6 \cdot 6^5 \cdot 6^8)^5 : (60^9 : 10^9)^7 \right]^6 : (6^5)^8 \right\}^9 : (2^5 \cdot 8^3 : 4^4)^3 \cdot 125^6$$

$$\text{II)} (\sqrt{x} - \sqrt{y})^2 - (\sqrt{x} - 1)(\sqrt{x} - \sqrt{y}) + (\sqrt{y} + 2)(\sqrt{x} + 1) - 3(y + \sqrt{x})$$

$$\text{I)} \left\{ \left[(6 \cdot 6^5 \cdot 6^8)^5 : (60^9 : 10^9)^7 \right]^6 : (6^5)^8 \right\}^9 : (2^5 \cdot 8^3 : 4^4)^3 \cdot 125^6 =$$

$$= \left\{ \left[(6^{1+5+8})^5 : \left(\left(\frac{60}{10} \right)^9 \right)^7 \right]^6 : 6^{5 \times 8} \right\}^9 : (2^5 \cdot (2^3)^3 : (2^2)^4)^3 \cdot 125^6 =$$

$$= \left\{ \left[(6^{14})^5 : (6^9)^7 \right]^6 : 6^{40} \right\}^9 : (2^5 \cdot 2^{3 \times 3} : 2^{2 \times 4})^3 \cdot 125^6 =$$

$$= \left\{ \left[6^{14 \times 5} : 6^{9 \times 7} \right]^6 : 6^{40} \right\}^9 : (2^5 \cdot 2^9 : 2^8)^3 \cdot (5^3)^6 =$$

$$= \left\{ \left[6^{70} : 6^{63} \right]^6 : 6^{40} \right\}^9 : (2^{5+9} : 2^8)^3 \cdot 5^{3 \times 6} =$$

$$= \left\{ \left[6^{70-63} \right]^6 : 6^{40} \right\}^9 : (2^{14} : 2^8)^3 \cdot 5^{18} =$$

$$= \left\{ [6^7]^6 : 6^{40} \right\}^9 : (2^{14-8})^3 \cdot 5^{18} =$$

$$= \left\{ 6^{7 \times 6} : 6^{40} \right\}^9 : (2^6)^3 \cdot 5^{18} =$$

$$= \left\{ 6^{42} : 6^{40} \right\}^9 : 2^{6 \times 3} \cdot 5^{18} =$$

$$= \left\{ 6^{42-40} \right\}^9 : 2^{18} \cdot 5^{18} =$$

$$= \left\{ 6^2 \right\}^9 : 2^{18} \cdot 5^{18} =$$

$$= 6^{2 \times 9} : 2^{18} \cdot 5^{18} =$$

$$= 6^{18} : 2^{18} \cdot 5^{18} =$$

$$= \left(\frac{6}{2} \right)^{18} \cdot 5^{18} =$$

$$= 3^{18} \cdot 5^{18} =$$

$$= (3 \cdot 5)^{18} =$$

$$= 15^{18}$$

$$\text{II) } (\sqrt{x} - \sqrt{y})^2 - (\sqrt{x} - 1)(\sqrt{x} - \sqrt{y}) + (\sqrt{y} + 2)(\sqrt{x} + 1) - 3(y + \sqrt{x})$$

$$= (\cancel{\sqrt{x}})^2 + (\cancel{\sqrt{y}})^2 - 2\sqrt{x}\sqrt{y} - [(\cancel{\sqrt{x}})^2 - \sqrt{x}\sqrt{y} - \sqrt{x} + \sqrt{y}] + \sqrt{y}\sqrt{x} + \sqrt{y} + 2\sqrt{x} + 2 - 3y - 3\sqrt{x} =$$

$$= \cancel{x} + \underline{y} + 2\sqrt{x}\sqrt{y} - \cancel{x} + \sqrt{x}\sqrt{y} + \underline{\underline{\sqrt{x}}} - \cancel{\sqrt{y}} + \sqrt{y}\sqrt{x} + \sqrt{y} + \underline{\underline{2\sqrt{x}}} + \underline{\underline{-6}}$$

$$+2 - 3y - 3\sqrt{x} =$$

$$= y(1-3) + \sqrt{xy}(-2+1+1) + \sqrt{x}(1+2-3) + 2 =$$

$$= y(-2) + \sqrt{xy}(0) + \sqrt{x}(0) + 2 =$$

$$= -2y + 0 + 0 + 2 =$$

$$= -2y + 2 =$$

$$= 2(1-y)$$