HOMEWORK 2
GAME THEORY PhD 2022

January 22, 2022

1 Exercise 1

Consider the extensive form games in figure 1

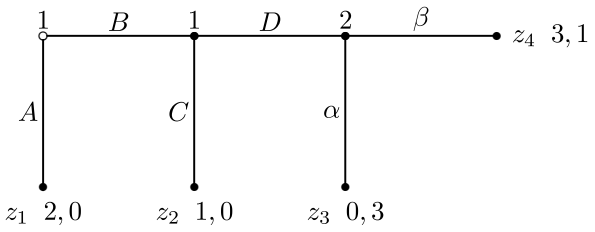

Figure 1

Calculate

1. the set of all Nash equilibria and the probabilities of outcomes in each of the equilibria;
2. the set of all subgame perfect equilibria, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria;
3. the set of weak Perfect Bayesian equilibria in pure strategies, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria;
4. the set of sequential equilibria in pure strategies, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria.

2 Exercise 2

Consider the extensive form game pictured in figure 2

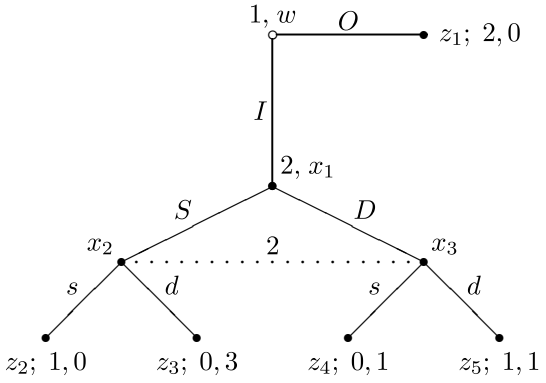

Figure 2

Calculate

1. the set of all subgame perfect equilibria, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria;

2. the set of weak Perfect Bayesian equilibria in pure strategies, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria;

3. the set of sequential equilibria in pure strategies, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria.

3 Exercise 3

Consider the game of figure 3

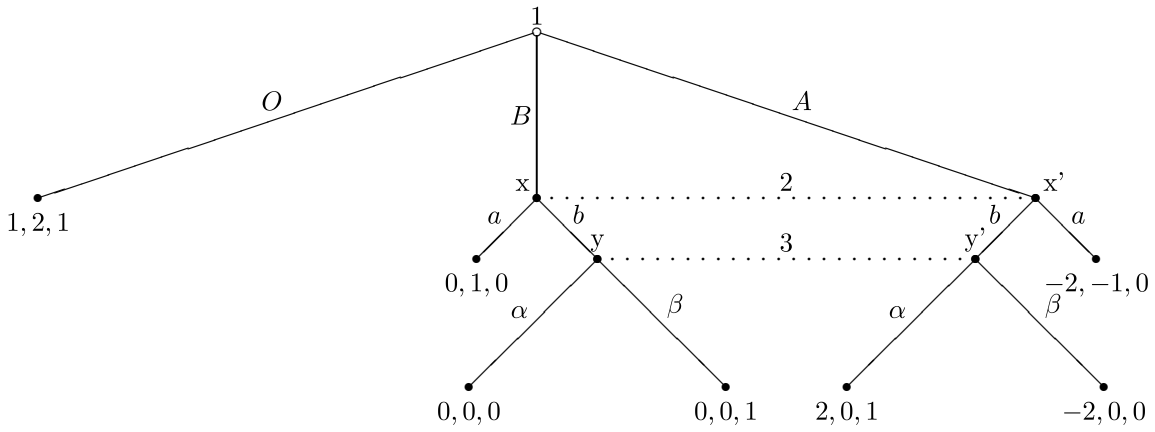

Figure 3

1. Construct the strategic form and calculate the set of Nash equilibria in pure strategies;
2. Calculate the set of Subgame Perfect Equilibria in pure strategies;
3. Calculate the set of Weak Perfect Bayesian Equilibria in pure strategies;
4. Find the set of sequential equilibria in pure strategies, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria;
5. Discuss the beliefs associated to each Sequential Equilibrium you find.

4 Exercise 4

Consider the game of figure 4

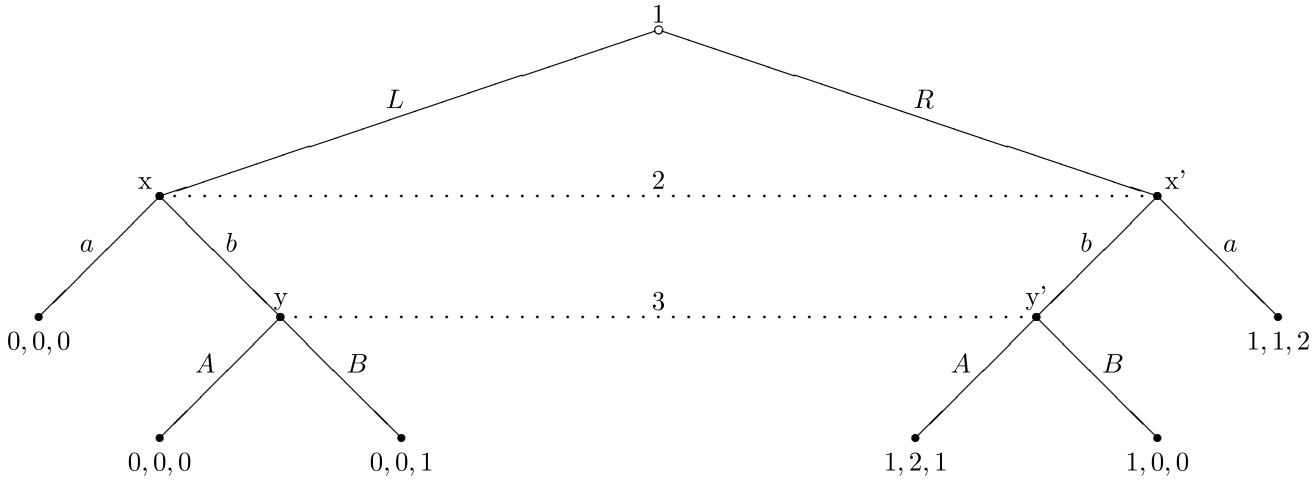

Figure 4

1. Construct the strategic form and calculate the set of Nash equilibria, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria;

2. Calculate the set of Subgame Perfect Equilibria, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria;

3. Calculate the set of Weak Perfect Bayesian Equilibria in pure strategies, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria;

4. Find the set of Sequential Equilibria in pure strategies, the probabilities of outcomes, emphasizing the equilibrium path and the out of equilibrium actions in each of the equilibria;

5. Discuss the beliefs associated to each WPBE.

5 Exercise 5

Before doing economics at the PhD in Economics at Bicocca you were a famous pastry cook. Mario knows this and regularly asks you to make cakes for him in exchange for good grades.

- This time, Mario needs **two cakes**;
- He will give you **one extra point on your final grade for each cake that he judges delicious**.

As usual, Mario made his request too late and you will not have time to make both cakes. However, you know a friend you can ask and who actually already helped you before to make cakes for Mario. Of course, this friend's cakes are usually less good than yours. In particular, you and your friend know that in the past,

- Matteo judged your cakes delicious 2/3 of the time while he judged your friend's cakes delicious only 1/3 of the time;
- your abilities are independent and you have the same production cost, normalized to 0;
- the two cakes are delivered in time but **Mario only gives you 1 extra point to share with your friend**;
- you set up a bargaining game to share this point.

The rules are the following:

1. you make an offer $s_Y \in [0; 1]$ which states that you want s_Y point for yourself and that your friend can keep $1 - s_Y$ point for himself;
2. your friend can then accept or reject this offer
   (a) if (s)he accepts it, (s)he gets $1 - s_Y$ point and you get s_Y point;
   (b) if (s)he rejects, then you have to bother Mario in his office and ask him directly. Mario will then give the point to the one that indeed cooked the delicious cake but will also remove $c = 1/5 + \varepsilon$ point to each of you because he does not like to be disturbed (with $\varepsilon > 0$ very small). Thus, if you end up going to Mario, the one that indeed made the cake will get $1 - c$ extra point on his grade and the other will actually loose c point;
3. finally, note that you and your friend have the same very simple utility function, defined over point won: $u(Points) = Points$.

QUESTIONS

1. Construct the extensive form game corresponding to this strategic situation;

2. show that the probability that you cooked the delicious cake is 4/5;

3. what are your expected utilities if your friend rejects your offer and you both end up going to Mario?

4. Is it possible to find Nash equilibria where you end up going to Mario's office?

5. Show that

$$\forall \alpha \in \left[\frac{3}{5}, 1\right] \quad \left(s_Y = \alpha, s_F\left(s_Y\right) = \left\{ \begin{array}{ccc} yes & \text{if} & s_Y \leq \alpha \\ No & \text{if} & s_Y > \alpha \end{array} \right.\right)$$

   is a Nash equilibrium. Note that, s_Y denotes your strategy and $s_F\left(s_Y\right)$ denotes your friend's strategy. What are the corresponding payoffs?

6. What are the subgame perfect Nash equilibria of the game?

7. Consider what happens if instead of the game ending after the decision of your friend, it is extended to two periods: if (s)he rejects then the game moves to period $t = 2$, and (s)he has to make an offer $s_F \in [0; 1]$, which specifies that (s)he keeps s_F point for himself and let you $1 - s_F$ point. Then, you can accept or reject this offer. If you reject, you both go to Mario and your payoffs are defined as before and the game ends. Otherwise, as before you get $1 - s_F$ and your friend gets s_F. You have the same discount factor given by $\delta \in (0, 1)$. Find the subgame perfect Nash equilibria for this extension of the game.