Introduction to Global Games

Lecture 02

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A General Approach to Symmetric Binary Action Global Games with a Continuum of Players

Uniform Prior
&
Private Values

Uniform Prior and Private Values - 1

- **Model:**

1. There is a continuum of players $i \in [0,1]$
2. Each player has to choose an action $a \in \{0,1\}$
3. All players have the same payoff function $u : \{0,1\} \times [0,1] \times \mathbb{R} \to \mathbb{R}$ where $u(a, l, \theta_i)$ is i's player's payoff if she chooses action a, a proportion l of the other players choose action 1, and her "private signal" is θ_i.

- **Information structure:**

1. $\theta \sim U(R)$
2. $\theta_i = \theta + \sigma\varepsilon_i$ with $\sigma > 0$
3. ε_i is a noise distributed on R according to a **continuous density f(·)** , possibly non symmetric and with mean different from 0.

- **Result**: the density of $\theta | \theta_i$ is well defined and is $\left(\frac{1}{\sigma}\right) f\left(\frac{\theta_i - \theta}{\sigma}\right)$

Uniform Prior and Private Values - 2

- **Remark: since** i's payoff is independent of which of the opponents choose action 1, to analyze best responses, it is enough to know the payoff gain from choosing one action rather than the other. Thus, the utility function is parameterized by a function

$$\pi : [0,1] \times \mathbb{R} \to \mathbb{R} \textit{ such that } \pi(l, \theta_i) \equiv u(1, l, \theta_i) - u(0, l, \theta_i).$$

- **Definition:** An action is the Laplacian action if it is a best response to a uniform prior over the opponents' choice of action.

Uniform Prior and Private Values - 3

Assumption 2 *The players' payoffs satisfy the following properties*

P.1 ***Action Monotonicity**: $\pi(l, \theta)$ is nondecreasing in l, i.e. the players actions are strategic complements;*

P.2 ***State Monotonicity**: $\pi(l, \theta)$ is nondecreasing in θ, i.e. a player's optimal action is increasing in the unknown state;*

P.3 ***Strict Laplacian State Monotonicity**: there exists a unique θ^* solving $\int_{l=0}^{1} \pi(l, \theta^*)dl = 0$, i.e. there is at most one crossing for a player with Laplacian beliefs;*

P.4 ***Limit Dominance**: there exist $\underline{\theta} \in \mathbb{R}$ and $\overline{\theta} \in \mathbb{R}$, such that*

(a) $\pi(l, \theta_i) < 0$ *for all* $l \in [0, 1]$ *and for all* $\theta_i \leq \underline{\theta}$, *i.e. action* $a = 0$ *is a dominant strategy for sufficiently low signals;*

(b) $\pi(l, \theta_i) > 0$ *for all* $l \in [0, 1]$ *and for all* $\theta_i \geq \overline{\theta}$, *i.e. action* $a = 1$ *is a dominant strategy for sufficiently high signals;*

P.5 ***Continuity**: $\int_{l=0}^{1} g(l)\, \pi(l, \theta_i)dl$ is continuous with respect to θ_i and density g with respect to the weak topology, therefore the payoff function might be discontinuous at one value of l.*

Uniform Prior and Private Values - 4

- **Definition**: Let define the game satisfying these assumptions as **$G^*(\sigma)$**.

- **Proposition:** In game $G^*(\sigma)$ there is essentially a **unique iterated strictly undominated strategy profile** $(s_i^*)_{i\in[0,1]}$ such that

$$\forall i \in [0,1] \qquad s_i^*(\theta_i) = \begin{cases} 0\, if & \theta_i < \theta^* \\ 1\, if & \theta_i > \theta^* \end{cases} \quad where \quad \theta^* \ satisfies \int_0^1 \pi(l, \theta^*)\, dl = 0.$$

- **Sketch of the proof:** The key idea of the proof is that, with a uniform prior on θ, observing θ_i gives no information to a player on her ranking within the population of signals. Thus, she will have a uniform belief over the proportion of players who will observe higher signals.

Comments

- Assumptions P.1 and P.2 represent very strong monotonicity assumptions:
- P.1 requires that each player's utility function is supermodular in the action profile,
- P.2 requires that each player's utility function is supermodular in his own action and the state.
- Vives (1990) showed that the supermodularity property P.2 of complete information game payoffs is inherited by the incomplete information game.
- Thus, the existence of a largest and smallest strategy profile surviving iterated deletion of dominated strategies when payoffs are supermodular, noted by Milgrom and Roberts (1990), can be applied also to the incomplete information game.
- The state monotonicity assumption P.2 implies, in addition, that the largest and smallest equilibria consist of cutoff strategies.
- Once we know that we are interested in cutoff strategies, the very weak assumption P.3 is sufficient to ensure the equivalence of the largest and smallest equilibria and thus the uniqueness of equilibrium.

General Prior
&
Common Values

General Prior and Common Values - 1

- **Model:**

1. There is a continuum of players $i \in [0,1]$
2. Each player has to choose an action $a \in \{0,1\}$
3. All players have the same payoff function $u : \{0,1\} \times [0,1] \times \mathbb{R} \to \mathbb{R}$ where $u(a, l, \boldsymbol{\theta})$ is i's player's payoff if she chooses action a, a proportion *l* of the other players choose action 1, and the realized state is $\boldsymbol{\theta}$.

- **Information structure:**

1. $\theta \sim p(R)$ where p(R) is a a continuously differentiable strictly positive density on the real line R.
2. $\theta_i = \theta + \sigma\varepsilon_i$ with $\sigma > 0$
3. ε_i is a noise distributed on R according to a continuous density $f(\cdot)$, possibly non symmetric and with mean different from 0.
4. $\int_{-\infty}^{\infty} zf(z)dz$ is well defined.

General Prior and Common Values - 2

- We must impose two extra technical assumptions:

Assumption 3 *P.4* Uniform Limit Dominance: there exist* $\underline{\theta} \in \mathbb{R}$, *a* $\overline{\theta} \in \mathbb{R}$, *and a strictly positive* $\epsilon \in \mathbb{R}_{++}$ *such that*

$$(a)\ \pi(l,\theta) < -\epsilon \ \text{ for all } l \in [0,1] \ \text{ and for all } \theta \leq \underline{\theta};$$

$$(b)\ \pi(l,\theta) > \epsilon \ \text{ for all } l \in [0,1] \ \text{ and for all } \theta \geq \overline{\theta}.$$

- **Remark**: Assumption P.4* strengthens assumption P.4 of Limit Dominance by requiring that the payoff gain to choosing action 0 is uniformly negative for sufficiently low values of θ, and the payoff gain to choosing action 1 is uniformly positive for sufficiently high values of θ.

General Prior and Common Values - 3

- **Definition**: Let define the game satisfying assumptions P.1, P.2, P.3, P.4*, P.5 and I.6 as **G(σ)**.

- **Proposition:** Let θ^* be defined solving $\int_0^1 \pi(l,\theta^*)dl=0$. For any $\delta>0$, there exists $\underline{\sigma}>0$ such that for all $\sigma \geq \underline{\sigma}$, if strategy s_i **survives iterated deletion of strictly dominated strategies** in the game G(σ), then

$$\forall i \in [0,1] \qquad s_i(\theta_i) = \begin{cases} 0 \; if & \theta_i < \theta^* - \delta \\ 1 \; if & \theta_i > \theta^* + \delta. \end{cases}$$

Inefficiency of Equilibrium Outcomes in Global Games

Inefficiency of Equilibrium Outcomes in Global Games

- **Result:** In general, in global games the equilibrium outcomes are not efficient.
- **Proof:**
- In equilibrium all players will be choosing action 1 when the state is θ if

$$\int_0^1 \pi(l,\theta)\, dl > 0.$$

- On the other hand, efficiency requires to choose action 1 at state θ if u(1,1,θ) > u(0,0,θ), and these conditions will not coincide in general.
- For **example**, in the investment game, we had π(l,θ) = u(1,l,θ) - u(0,l,θ) = θ+l-1 that implies

$$\int_0^1 \pi(l,\theta)\, dl = \int_0^1 (\theta + l - 1) = \theta - \frac{1}{2}.$$

so that the players will be investing if the state θ is at least (1/2), although it is efficient for them to be investing if the state is at least 0.

Public and Private Signals

Public and Private Signals - 1

- To understand the effects of public signals for global games, consider the Investment Game with a continuum of players previously analyzed with private information only.
- **Example:**
- There is a continuum of players $i \in [0,1]$
- who should decide whether to invest or not.
- The payoff is

$$U_i(L,l) = \begin{cases} \theta + l - 1 & L = Invest \\ 0 & L = Not \quad Invest \end{cases}$$

where l is the proportion of other players choosing to invest.

- The **information structure** is:
- each player i observes a private signal $\theta_i = \theta + \sigma\varepsilon_i$
- where ε_i is identically and independently normally distributed $\varepsilon_i \sim N(0,1)$
- $\theta \sim N(y, \tau)$ where **y is a public signal**.

Public and Private Signals - 2

- From standard statistics (e.g. De Groot 1970), we get the following result.
- **Result:**

$$E(\theta|\theta_i) = \frac{\sigma^2 y + \tau^2 \theta_i}{\sigma^2 + \tau^2}$$

- Consider the following cutoff strategy

$$s(E(\theta|\theta_i)) = \begin{cases} Invest & \text{if } E(\theta|\theta_i) > \kappa \\ Not\ \ Invest & \text{if } E(\theta|\theta_i) \leq \kappa. \end{cases}$$

- Let define

$$\widetilde{\gamma}(\sigma, \tau) = \frac{\sigma^2}{\tau^4}\left(\frac{\sigma^2 + \tau^2}{\sigma^2 + 2\tau^2}\right).$$

Public and Private Signals - 3

- Morris and Shin (2006) prove the following result.
- **Proposition:** The game has a symmetric switching strategy equilibrium with cutoff κ if κ solves the equation

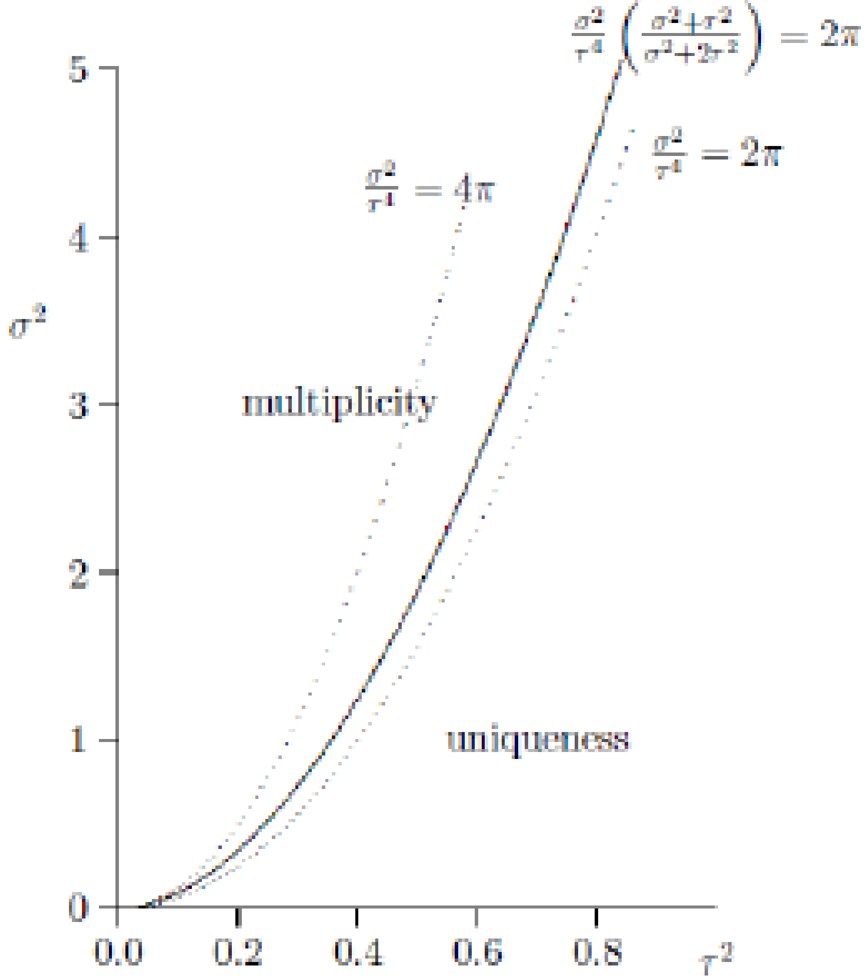
$$\kappa = \Phi\left(\sqrt{\widetilde{\gamma}}\,(\kappa - y)\right);$$

then

1. if γ(σ,τ)≤2π, there is a unique value of κ solving the previous equation and the strategy with cutoff κ is the essentially unique strategy surviving iterated deletion of strictly dominated strategies;
2. if γ(σ,τ)>2π,, then (for some values of y) there are multiple values of κ solving the previous equation and multiple symmetric cutoff strategy equilibria.

Public and Private Signals - 4

- The following picture plots the two regions in the space (τ^2, σ^2):



Public and Private Signals - 5

- **Corollary:** Suppose $\gamma(\sigma,\tau) \leq 2\pi$, then

1. if $E(\theta|\theta_i) < 0$, in equilibrium for any y it is optimal not to invest;
2. if $E(\theta|\theta_i) > 1$, in equilibrium for any y it is optimal to invest;
3. if $E(\theta|\theta_i) \in [0,1]$, then in equilibrium the higher y, the more likely it is optimal to invest.
    - Thus, the players will always invest for sufficiently high y, and not invest for sufficiently low y.
    - This implies that **changing y has a larger impact on a player's action than changing his private signal (controlling for the informativeness of the signals), the "publicity" effect**.

The Role of Public and Private Signals

The Role of Public and Private Information - 1

- To explore the strategic impact of public information, we examine how much a player's private signal must adjust to compensate for a given change in the public signal.
- Consider the cutoff equation $\kappa = \Phi\left(\sqrt{\tilde{\gamma}}\,(\kappa - y)\right)$ when $\kappa = \overline{\theta} = \frac{\sigma^2 y + \tau^2 \theta_i}{\sigma^2 + \tau^2}$,
- i.e. κ is equal to the expected value of $\theta | \theta_i$), so that a player is indifferent between investing and not investing, which implies

$$\frac{\sigma^2 y + \tau^2 \theta_i}{\sigma^2 + \tau^2} = \Phi\left(\sqrt{\tilde{\gamma}}\left(\frac{\sigma^2 y + \tau^2 \theta_i}{\sigma^2 + \tau^2} - y\right)\right)$$

- Considering this equation as an implicit function $\theta_i(y)$, we have

$$\frac{d\theta_i(y)}{dy} = -\frac{\frac{\sigma^2}{\tau^2} + \sqrt{\tilde{\gamma}}\phi(\cdot)}{1 - \sqrt{\tilde{\gamma}}\phi(\cdot)}$$

- which measures **how much the private signal would have to change to compensate for a change in the public signal** **leaving the player indifferent between investing or not investing**.

The Role of Public and Private Information - 2

- On the other hand, if we ignore strategic effect of a change in y, the private signal has a different substitution ratio that can be derived maintaining constant
  $\bar{\theta} = \frac{\sigma^2 y + \tau^2 \theta_i}{\sigma^2 + \tau^2}$ so that $\frac{d\theta_i}{dy} = -\frac{\sigma^2}{\tau^2}.$

- The ratio between these two substitution ratio defines the **publicity multiplier**

$$\zeta = \frac{1 + \frac{\tau^2}{\sigma^2}\sqrt{\tilde{\gamma}}\phi(\cdot)}{1 - \sqrt{\tilde{\gamma}}\phi(\cdot)}$$

- which is **increasing in gamma tilde**, which is intuitive since it is directly related to the informativeness of the public versus the private signal.
- However, remember that gamma tilde is bounded above by 2π, otherwise we go into the regions of multiple equilibria.

<u>Applications</u>:

a quick survey of some simple basic models

Pricing Debt - 1

- This application refers to Morris and Shin (2004). Consider the following simple model.
  1. There are two periods:
     1. in period 1, a continuum of investors hold collateralized debt that will pay
        - 1 in period 2 if it is rolled over and if an underlying investment project is successful;
        - 0 in period 2 if the project is not successful;
        - $\kappa \in (0,1)$, the value of the collateral, if an investor does not roll over his debt.
  2. The success of the project depends on
     - the proportion l of investors who do not roll over and
     - the state of the economy, θ, which is distributed according to a continuum density $p(\cdot)$
     - Specifically, the project is successful if the proportion of investors not rolling over is less than θ/z.

Pricing Debt - 2

3. Write a=1 for the action "roll over" and a=0 for the action "do not roll over", then the payoffs can be written as follows:

$$u(a,l,\theta) = \begin{cases} 1 \text{ if } & a = 1 \text{ and } \frac{\theta}{z} \geq 1-l \\ 0 \text{ if } & a = 1 \text{ and } \frac{\theta}{z} < 1-l \\ \kappa & \text{if } a = 0 \end{cases}$$

or alternatively

		Percentage of investors rolling over	
		$1-l \leq \frac{\theta}{z}$	$1-l > \frac{\theta}{z}$
$i \in [0,1]$	$a=1$	1	0
	$a=0$	κ	κ

Payoff structure

Hence,

$$\pi(l,\theta) = u(1,l,\theta) - u(0,l,\theta) = \begin{cases} 1-\kappa \text{ if } & \frac{\theta}{z} \geq 1-l \\ -\kappa \text{ if } & \frac{\theta}{z} < 1-l \end{cases}$$

so that

$$\int_0^1 \pi(l,\theta)\,dl = \begin{cases} -\kappa & \text{if } \theta \leq 0 \\ \frac{\theta}{z} - \kappa & \text{if } 0 \leq \theta \leq z \\ 1-\kappa & \text{if } \theta \geq z. \end{cases}$$

Pricing Debt - 3

Remark: The game representing the model satisfies assumptions P.1* and P.2, and therefore Proposition 3 holds.

Thus, we can state the following result.

Result: Let $\theta^* = z\kappa$, then the game has a unique (symmetric) cutoff strategy equilibrium, such that

$$\forall i \in [0,1] \qquad s_i(\theta_i) = \begin{cases} 0 \; if & \theta_i \leq \theta^* \\ 1 \; if & \theta_i > \theta^*. \end{cases}$$

Remark: In other words, if private information about θ among the investors is sufficiently accurate, the project will collapse if $\theta \leq z\kappa$.

Question: We can now ask how debt would be priced ex ante in this model, i.e. before anyone observed private signals about θ.

Recalling that p(·) is the density of the prior on θ, and writing P(·) for the corresponding cdf, the value of the collateralized debt will be

$$V(\kappa) \equiv \kappa P(z\kappa) + 1 - P(z\kappa) = 1 - (1-\kappa) P(z\kappa)$$

that implies

$$\frac{dV(\kappa)}{d\kappa} = P(z\kappa) - z(1-\kappa)p(z\kappa).$$

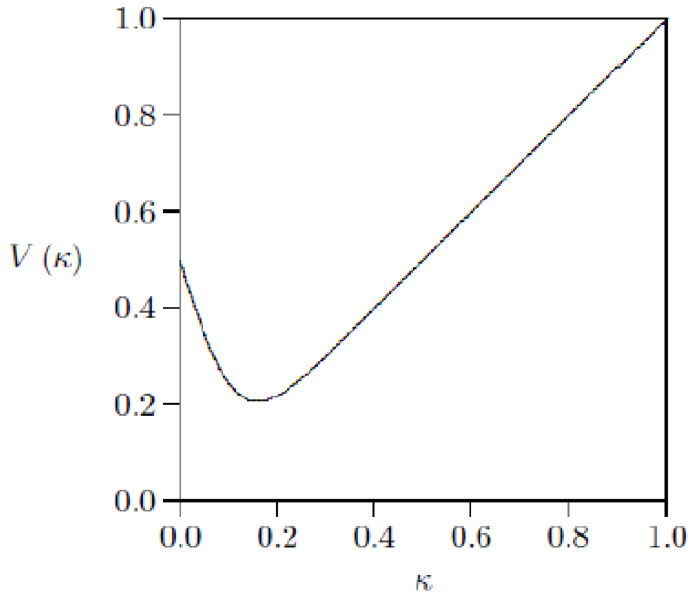
Pricing Debt - 4

Since $$\frac{dV(\kappa)}{d\kappa} = P(z\kappa) - z(1-\kappa)p(z\kappa).$$

then increasing the value of collateral has two effects:

1. it increases the value of debt in the event of default (the **direct effect**);
2. it increases the range of θ at which default occurs (the **strategic effect**).

- For small κ, the strategic effect outweighs the direct effect, whereas for large κ, the direct effect outweighs the strategic effect.
- The following figure plots V(κ) for the case where z=10 and p(·) is the standard normal density.



Currency Crises - 1

- This application refers to Morris and Shin (1998). Consider the following simple **model**.
  1. There is a continuum of speculators $i \in [0, 1]$ must decide whether to attack a fixed-exchange rate regime by selling the currency short.
  2. Each speculator may short only a unit amount.
  3. There is a fixed transaction cost t of attacking, that can be interpreted as an actual transaction cost or as the interest rate differential between currencies.
  4. The current value of the currency is e^*;
  5. the monetary authority may defend or not the currency
     1. if the monetary authority does not defend the currency, the currency will float to the shadow rate $\zeta(\theta)$, where θ is the state of fundamentals, so that $\zeta(\theta)$ is increasing in θ. Assume $\zeta(\theta) \leq e^* - t$ for all θ;
     2. if the monetary authority does defend the currency, its value remains at e^*
     3. The monetary authority defends the currency if the cost of doing so is not too large, where the costs of defending the currency are
        - increasing in the proportion of speculators who attack and
        - decreasing in the state of fundamentals.
     4. Hence, there will be a critical proportion of speculators, $b(\theta)$, increasing in θ, who must attack in order for a devaluation to occur.

Currency Crises - 2

6. Write a=1 for the action "not attack" and a=0 for the action "attack", then the payoffs can be written as follows:

$$u(a,l,\theta) = \begin{cases} 0 & \text{if } a=1 \\ e^* - \zeta(\theta) - t & \text{if } a=0 \text{ and } 1-l \geq b(\theta) \\ -t & \text{if } a=0 \text{ and } 1-l < b(\theta) \end{cases}$$

or alternatively

		Percentage of attacking speculators	
		$1-l < b(\theta)$	$1-l \geq b(\theta)$
$i \in [0,1]$	$a=1$	0	0
	$a=0$	$-t$	$e^* - \zeta(\theta) - t$

Payoff structure

Hence,

$$\pi(l,\theta) = u(1,l,\theta) - u(0,l,\theta) = \begin{cases} \zeta(\theta) + t - e^* & \text{if } l \leq 1-b(\theta) \\ t & \text{if } l \leq 1-b(\theta). \end{cases}$$

Currency Crises - 3

- **Result:** Suppose θ is common knowledge, then
  1. if $\theta < b^{-1}(0)$, then there is unique equilibrium in dominant strategies, $a^* = 0$ for all $i \in [0,1]$;
  2. if $b^{-1}(0) \leq \theta \leq b^{-1}(1)$, then there two equilibria such that $a^* = 0$ for all $i \in [0,1]$ and $a^{**} = 1$ for all $i \in [0,1]$;
  3. if $\theta > b^{-1}(1)$, then there is unique equilibrium in dominant strategies, $a^* = 1$ for all $i \in [0,1]$.

- On the other hand, if θ is observed with noise, we can apply the previous results, because the previous assumptions are satisfied. In particular

$$\int_0^1 \pi(l,\theta)\,dl = [1-b(\theta)][\zeta(\theta)+t-c^*] + b(\theta)\,t$$

which implies $\int_0^1 \pi(l,\theta^*)\,dl = 0 \Leftrightarrow [1-b(\theta^*)][\zeta(\theta^*)-c^*] = t$. Thus, we can state the following result

- **Result**: The game representing our model with private information on θ has a unique (symmetric) cutoff strategy equilibrium

$$\forall i \in [0,1] \qquad s_i(\theta_i) = \begin{cases} 0 \; if & \theta_i \leq \theta^* \\ 1 \; if & \theta_i > \theta^*. \end{cases}$$

Bank Runs - 1

- Consider the following simple model by Goldstein and Pauzner (2005), who add noise to the classic bank runs model of Diamond and Dybvig (1983).
  1. There are two periods, 1 and 2
  2. There is a continuum of depositors $i \in [0,1]$ (with total deposits normalized to 1)
  3. Each depositor must decide whether to withdraw their money from a bank at period 1, denoted $a=0$, or at period 2, denoted by $a=1$.
  4. The withdrawn resources are entirely used for consumption that gives utility $U(\cdot)$.
  5. A proportion λ of depositors will have consumption needs only in period 1 and will thus have a dominant strategy to withdraw, thus we are concerned with the game among the proportion $1-\lambda$ of depositors.

Bank Runs - 2

6. The monetary payoffs are:
   - $r>1$ if the depositors withdraw their money in period 1 and there are enough resources;
   - $\frac{1-\lambda r}{(1-l)(1-\lambda)}$ if there are not enough resources to fund all those who try to withdraw – which are $1-l$, i.e. the remaining cash $1-\lambda r$ is divided equally among early withdrawers. This happens when

$$\lambda r + (1-l)(1-\lambda) r \geq 1 \Leftrightarrow l \leq \frac{r-1}{(1-\lambda) r};$$

   - $\max\{0, 1-\lambda r+(1-l)(1-\lambda)r\}R(\theta) \geq 0$ in period 2 for those who chose to wait until period 2 to withdraw their money, i.e. any remaining money after period 1 withdraws, $\max\{0, 1-\lambda r+(1-l)(1-\lambda)r\}$, earns a total return $R(\theta)>0$ in period 2, which is increasing in θ, and it is divided equally among those who chose to wait until period 2 to withdraw their money, $l(1-\lambda)$.

Bank Runs - 3

Then the consumers monetary payoffs can be written as follows:

		Percentage of late consumers	
		$l \leq \frac{r-1}{(1-\lambda)r}$	$l \geq \frac{r-1}{(1-\lambda)r}$
$i \in [0,1]$	$a=0$	$\frac{1-\lambda r}{(1-l)(1-\lambda)}$	r
	$a=1$	0	$\left[\frac{1-\lambda r+(1-l)(1-\lambda)r}{l(1-\lambda)}\right] R(\theta)$

Monetary payoff structure

Thus, the utilities of late consumers are

$$u(a,l,\theta) = \begin{cases} U\left(\frac{1}{1-l(1-\lambda)}\right) & \text{if } a=0 \text{ and } l \leq \frac{r-1}{(1-\lambda)r} \\ U(r) & \text{if } a=0 \text{ and } l \geq \frac{r-1}{(1-\lambda)r} \\ U(0) & \text{if } a=1 \text{ and } l \leq \frac{r-1}{(1-\lambda)r} \\ U\left(\left[\frac{1-\lambda r+(1-l)(1-\lambda)r}{l(1-\lambda)}\right] R(\theta)\right) & \text{if } a=0 \text{ and } l \geq \frac{r-1}{(1-\lambda)r} \end{cases}$$

Hence,

$$\pi(l,\theta) = u(1,l,\theta) - u(0,l,\theta) = \begin{cases} U(0) - U\left(\frac{1}{1-l(1-\lambda)}\right) & \text{if } l \leq \frac{r-1}{(1-\lambda)r} \\ U\left(\left[\frac{1-\lambda r+(1-l)(1-\lambda)r}{l(1-\lambda)}\right] R(\theta)\right) - U(r) & \text{if } l \geq \frac{r-1}{(1-\lambda)r} \end{cases}$$

Bank Runs - 4

- **Result:** Suppose θ is common knowledge, then for late consumers
  1. if θ is small so that also R(θ) is small, then there is unique equilibrium in dominant strategies, $a^* = 0$;
  2. if θ is intermediate so that also R(θ) is intermediate, then there are two equilibria, $a^* = 0$ and $a^{**} = 1$ for all $i \in [0,1]$;
  3. if θ is large so that also R(θ) is large, then there is unique equilibrium in dominant strategies, $a^* = 1$.

Bank Runs - 5

- On the other hand, if θ is observed with noise, we can apply the previous results, because the previous assumptions are satisfied.
- **Remark:** the game representing the model satisfies assumptions P.1 and P.2, and therefore Proposition 2 holds. In particular θ^* is defined by the following

$$\int_0^1 \pi(l, \theta^*)\, dl = 0 \Leftrightarrow$$

$$\Leftrightarrow \int_0^{\frac{r-1}{(1-\lambda)r}} \left[U(0) - U\left(\frac{1}{1-l(1-\lambda)}\right)\right] dl +$$

$$+ \int_{\frac{r-1}{(1-\lambda)r}}^1 \left[U\left(\left[\frac{1-\lambda r + (1-l)(1-\lambda) r}{l(1-\lambda)}\right] R(\theta^*)\right) - U(r)\right] dl = 0.$$

- **Result:** The game representing our model with private information on θ has a unique (symmetric) cutoff strategy equilibrium, such that

$$s_i(\theta_i) = \begin{cases} 0 \; if \;\; \theta_i \le \theta^* \\ 1 \; if \;\; \theta_i > \theta^*. \end{cases}$$