Transparency of Information and Coordination in Economies with Investment Complementarities

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Research Question

How do public and private information affect equilibrium allocations and social welfare in economies with investment complementarities?
What is the optimal transparency in the information conveyed, for example by economic statistics, policy announcements, or news in media?

Introduction

- In this paper there are two environments, the first with weak complementarities, the second with strong complementarities.
- This contribute is interessing because became to different conclusion respect to Morris and Shin (2002).
- In Morris and Shine (2002) more precise public information can reduce social welfare, whereas more precise private information is always beneficial.
- We will see that in this paper the result is the opposite.

Weak Complementarities

The economy is populated by a continuum of measure one of agents, indexed by i and uniformly distributed over the $[0, 1]$ interval.

Agents are risk neutral with utility

$$u_i = Ak_i - \frac{1}{2}k_i^2;$$

$$K = \int_0^1 k_i di;$$

$$A = (1-\alpha)\theta + \alpha K;$$

with $\alpha \geq 0$.

Weak Complementarities

Social welfare is given by a utilitarian aggregator, $w = \int_0^1 u_i di$, using the previous conditions we can rewrite

$$w = \int_0^1 Ak_i - \frac{1}{2}k_i^2 di;$$

$$w = A\int_0^1 k_i di - \frac{1}{2}\int_0^1 k_i^2 di;$$

$$w = AK - \frac{1}{2}K^2 - \frac{1}{2}\int_0^1 (k_i - K)^2 di;$$

$$w = ((1-\alpha)\theta + \alpha K)K - \frac{1}{2}K^2 - \frac{1}{2}\int_0^1 (k_i - K)^2 di;$$

$$w = (1-\alpha)\theta K + \alpha K^2 - \frac{1}{2}K^2 - \frac{1}{2}\int_0^1 (k_i - K)^2 di;$$

$$w = (1-\alpha)\theta K - (1-2\alpha)\frac{1}{2}K^2 - \frac{1}{2}\int_0^1 (k_i - K)^2 di.$$

Weak complementarities

$$w = (1-\alpha)\theta K - (1-2\alpha)\frac{1}{2}K^2 - \frac{1}{2}var.$$

w is concave in K for $\alpha < \frac{1}{2}$ and convex for $\alpha > \frac{1}{2}$:

$$w'(K) = (1-\alpha)\theta - (1-2\alpha)k - 2K;$$

$$w''(K) = 2\alpha - 1.$$

Information and Trasparency

The fundamental $\theta \in \mathbb{R}$ are not known at the time investment decision are made. Common prior about θ is **uniform** over $\mathbb{R}$. The public information is:

$$z = \theta + \sigma_z \epsilon.$$

where ϵ is standard normal ($\epsilon \sim \mathcal{N}(0, 1)$), independent of θ and common across agents.
Private information

$$x_i = \theta + \sigma_x \xi_i.$$

where ξ_i is a standard normal, independent of θ and i.i.d. across agents.
In the model, σ_z and σ_x parametrize the precision of public and private information.

Model

The authors define

$$\delta \equiv \frac{\sigma_z^{-2}}{\sigma_x^{-2} + \sigma_z^{-2}};$$

and

$$\sigma \equiv \sqrt{\sigma_x^{-2} + \sigma_z^{-2}}.$$

The posterior belief of agent i about θ is normal with mean $\mathbb{E}_i[\theta] \equiv \mathbb{E}_i[\theta|x_i, z] = (1 - \delta)x_i + \delta z$ and variance $\text{Var}_i[\theta] \equiv \text{Var}_i[\theta|x_i, z] = \sigma^2$.

Model

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The posterior belief of agent i about θ is normal with mean $\mathbb{E}_i[\theta] \equiv \mathbb{E}_i[\theta | x_i, z] = (1-\delta)x_i + \delta z$ and variance $\text{Var}_i[\theta] \equiv \text{Var}_i[\theta | x_i, z] = \sigma^2$. We can rewrite δ in this way

$$1 - \delta = 1 - \frac{\sigma_z^{-2}}{\sigma_x^{-2} + \sigma_z^{-2}} \implies 1 - \delta \frac{\sigma_x^{-2}\sigma_z^2 + 1 - 1}{\sigma_x^{-2}\sigma_z^2};$$

$$1 - \delta = \frac{\sigma_x^{-2}\sigma_z^2}{\frac{\sigma_x^2 + \sigma_z^2}{\sigma_x^2}} \implies 1 - \delta = \frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2}.$$

Model

$$\delta = 1 - \frac{\sigma_z^2}{\sigma_x^2 + \sigma_z^2} + 1;$$

$$\delta = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2}.$$

We can see that $\sigma = \sqrt{\sigma_z^2 \delta}$. Thus,

$$\sigma = \sqrt{\frac{\sigma_z^2 \sigma_x^2}{\sigma_x^2 + \sigma_z^2}} = \frac{\sigma_z \sigma_x}{\sqrt{\sigma_x^2 + \sigma_z^2}};$$

$\mathbb{E}_i[\theta | x_i, z] = \frac{\sigma_z^2 x_i + \sigma_x^2 z}{\sigma_x^2 + \sigma_z^2}$ and $\text{Var}_i[\theta] \equiv \text{Var}_i[\theta | x_i, z] = \frac{\sigma_z^2 \sigma_x^2}{\sigma_x^2 + \sigma_z^2}$.

Equilibrium

Every agent chooses k_i so as to maximize $\mathbb{E}_i[u_i]$

$$k_i = \mathbb{E}_i[A] = (1-\alpha)\mathbb{E}_i[\theta] + \alpha\mathbb{E}_i[K].$$

Individual investment is **increasing** in the expected level of the fundamentals and in the expected level of aggregate investment. Given the linearity of expected utility and the normality of posterior beliefs about θ, equilibrium investment decisions are linear so that $k_i = \beta x_i + \gamma z$. Then $K = \int_0^1 (\beta x_i + \gamma z) di = \int_0^1 \beta\theta + \beta\sigma_x \xi_i di$,

$$K = \beta\theta + \gamma z + \beta\sigma_x \int_0^1 \xi_i di;$$

$$K = \beta\theta + \gamma z + \beta\sigma_x \int_0^1 \mathbb{E}_i[\xi_i] di = \beta\theta + \gamma z;$$

$$k_i = \mathbb{E}_i[A] = (1-\alpha)[(1-\delta)x_i + \delta z] + \alpha\beta[(1-\delta)x_i + \delta z] + \alpha\gamma z$$

;

$$k_i = (1-\alpha+\alpha\beta)[(1-\alpha)x_i + \delta z] + \alpha\gamma z.$$

Equilibrium

Now we can evaluate β and γ,

$$\beta x_i = (1 - \alpha + \alpha\beta)(1 - \delta)x_i;$$

$$\beta = (1 - \alpha + \alpha\beta)(1 - \delta);$$

$$\beta - \alpha\beta(1 - \delta) = (1 - \alpha)(1 - \delta);$$

$$\beta(1 - -\alpha(1 - \delta)) = (1 - \alpha)(1 - \delta);$$

$$\beta = \frac{(1 - \alpha)(1 - \delta)}{1 - \alpha(1 - \delta)}.$$

Equilibrium

$$\gamma z = (1 - \alpha + \alpha\beta)\delta z + \alpha\gamma z;$$

$$\gamma = (1 - \alpha + \alpha\beta)\delta + \alpha\gamma;$$

$$\gamma(1 - \alpha) = (1 - \alpha + \alpha\beta)\delta;$$

$$\gamma(1 - \alpha) = (1 - \alpha)\delta + \alpha\beta\delta;$$

$$\gamma(1 - \alpha) = (1 - \alpha)\delta + \alpha\frac{(1 - \alpha)(1 - \delta)}{1 - \alpha(1 - \delta)}\delta;$$

$$\gamma = \delta + \alpha\frac{(1 - \delta)}{1 - \alpha(1 - \delta)}\delta;$$

$$\gamma = \frac{\delta}{1 - \alpha(1 - \delta)}.$$

Equilibrium

As proved in Morris and Shin (2002), there do no exist equilibria other than $k_i = \beta x_i + \gamma z$.

Proposition 1 *The equilbrium exists, is unique, and is given by* $k_i = \beta x_i + \gamma z$, where

$$\beta = 1 - \delta - \rho, \quad \gamma = \delta + \rho \quad \rho = \frac{\alpha\delta(1-\delta)}{1-\alpha(1-\delta)}.$$

Equilibrium

The equilibrium level of volatility and heterogeneity are $Var(K|\theta) = (\gamma\sigma_z)^2$ and $Var(k_i|\theta, z) = (\beta\sigma_x)^2$, indeed, such that $\beta + \gamma = 1$, we have

$$Var(K|\theta) = (K - \theta)^2 = (K^2 - 2K\theta + \theta^2);$$

$$Var(K|\theta) = \left((\beta\theta + \gamma\theta + \gamma\sigma_z\epsilon)^2 - 2\theta(\beta\theta + \gamma\theta + \gamma\sigma_z\epsilon) + \theta^2\right);$$

$$Var(K|\theta) = \left(\theta^2 + 2\gamma\sigma_z\epsilon\theta + (\gamma\sigma\epsilon)^2 - 2\theta(\theta + \gamma\sigma_z\epsilon) + \theta^2\right);$$

$$Var(K|\theta) = \left((\theta^2 + 2\gamma\sigma_z\mathbb{E}[\epsilon]\theta + (\gamma\sigma)^2\mathbb{E}[\epsilon^2] - 2\theta^2 + \theta\gamma\sigma_z\mathbb{E}[\epsilon] + \theta^2\right);$$

$$Var(K|\theta) = (\gamma\sigma)^2.$$

Equilibrium

$$Var(k_i|\theta, z) = \int_0^1 (k_i - K)^2 di;$$

$$Var(k_i|\theta, z) = \int_0^1 (\beta x_i + \gamma z - \beta\theta - \gamma z)^2 di;$$

$$Var(k_i|\theta, z) = \int_0^1 (\beta x_i - \beta\theta)^2 di;$$

$$Var(k_i|\theta, z) = \int_0^1 (\beta(\theta + \sigma_x \xi_i) - \beta\theta)^2 di;$$

$$Var(k_i|\theta, z) = \int_0^1 (\beta\sigma_x \xi_i)^2 di;$$

$$Var(k_i|\theta, z) = (\beta\sigma_x)^2 \int_0^1 (\xi_i)^2 di;$$

$$Var(k_i|\theta, z) = (\beta\sigma_x)^2 \int_0^1 \mathbb{E}_i[\xi_i^2] di;$$

$$Var(k_i|\theta, z) = (\beta\sigma_x)^2 \cdot 1.$$

Equilibrium

Proposition 2 (i) *Volatility necessarily increases with an increase in* δ *for given* σ, *and increases with a reduction in* σ_z *for given* σ_x *if and only if* $\sigma_z^2 > \frac{1}{1-\alpha}\sigma_x^2$. **(ii)** *Heterogeneity falls with either an increase in* δ *or a reduction in* σ_z.

Equilibrium

Proof:

$$\frac{\partial Var(K|\theta)}{\partial \delta} = 2\sigma_z^2 \frac{\delta}{1-\alpha(1-\delta)} \cdot \frac{1-\alpha(1-\delta)-\alpha\delta}{[1-\alpha(1-\delta)]^2};$$

$$\frac{\partial Var(K|\theta)}{\partial \delta} = \frac{1-\alpha}{[1-\alpha(1-\delta)]^2} > 0;$$

$$Var(K|\theta) = (\gamma\sigma_z)^2 = \frac{\sigma_x^4\sigma_z^2}{[\sigma_x^2+(1-\alpha)\sigma_z^2]^2};$$

$$\frac{\partial Var(K|\theta)}{\partial \sigma_z} = \frac{2\sigma_z\sigma_x^4[\sigma_x^2+(1-\alpha)\sigma_z^2]^2 - 2[\sigma_x^2+(1-\alpha)\sigma_z^2]\cdot 2\sigma_z(1-\alpha)\sigma_x^4\sigma_z^2}{[\sigma_x^2+(1-\alpha)\sigma_z^2]^4} < 0;$$

$$\sigma_x^2 - \sigma_z^2(1-\alpha) < 0;$$

$$\sigma_z^2 > \frac{\sigma_x^2}{1-\alpha}.$$

Equilibrium

$$\frac{\partial Var(k_i|\theta,z)}{\partial \delta} = \frac{-2(1-\alpha)^2(1-\delta)[1-\alpha(1-\delta)]^2 - 2\alpha[1-\alpha(1-\delta)](1-\alpha)^2(1-\delta)^2}{[1-\alpha(1-\delta)]^4} < 0;$$

$$\frac{\partial Var(k_i|\theta,z)}{\partial \sigma_z} = \frac{4\sigma_z^3\sigma_x^2(1-\alpha)^2[\sigma_x^2+(1-\alpha)\sigma_z^2]^2 - 4(1-\alpha)\sigma_z[\sigma_x^2+(1-\alpha)\sigma_z^2][(1-\alpha)^2\sigma_z^4\sigma_x^2]}{[\sigma_x^2+(1-\alpha)\sigma_z^2]^4} > 0;$$

$$\frac{\partial Var(k_i|\theta,z)}{\partial \sigma_z} = \frac{\sigma_z^3\sigma_x^4}{[\sigma_x^2+(1-\alpha)\sigma_z^2]^4} > 0.$$

Welfare

Now, we have to compute the welfare. Thus, we have to evaluate

$$w(\theta) = \int_0^1 Ak_i - \frac{1}{2}k_i^2 di;$$

$$w(\theta) = \int_0^1 [(1-\alpha)\theta + \alpha K]k_i - \frac{1}{2}k_i^2 di;$$

$$w(\theta) = \int_0^1 [(1-\alpha)\theta + \alpha(\beta\theta + \gamma z)](\beta x_i + \gamma z) - \frac{1}{2}(\beta x_i + \gamma z)^2 di;$$

$$w(\theta) = \int_0^1 [(1-\alpha)\theta + \alpha(\theta + \gamma\sigma_z\epsilon)](\beta(\theta + \sigma_x\xi_i) + \gamma(\theta + \sigma_z\epsilon)) + \\ -\frac{1}{2}\Big(\beta(\theta + \sigma_x\xi_i) + + \gamma(\theta + \sigma_z\epsilon)\Big)^2 di.$$

Welfare

$$w(\theta) = \int_0^1 [\theta + \alpha\gamma\sigma_z\epsilon)](\theta + \beta\sigma_x\xi_i + \gamma\sigma_z\epsilon) - \frac{1}{2}(\theta + \beta\sigma_x\xi_i + \gamma\sigma_z\epsilon)^2 di;$$

$$w(\theta) = \int_0^1 \theta^2 + \theta\beta\sigma_x\xi_i + \theta\gamma\sigma_z\epsilon + \alpha\gamma\sigma_z\epsilon\theta + \alpha\gamma\sigma_z\epsilon\beta\sigma_x\xi_i + \alpha(\gamma\sigma_z\epsilon)^2 +$$
$$-\frac{1}{2}\left(\theta^2 + (\beta\sigma_x\xi_i)^2 + (\gamma\sigma_z\epsilon)^2 + 2\theta\beta\sigma_x\xi_i + 2\theta\gamma\sigma_z\epsilon + \beta\sigma_x\xi_i\gamma\sigma_z\epsilon\right) di$$

;

$$w(\theta) = \int_0^1 \theta^2 + \theta\beta\sigma_x\mathbb{E}_i[\xi_i] + \theta\gamma\sigma_z\mathbb{E}[\epsilon] + \alpha\gamma\sigma_z\mathbb{E}[\epsilon]\theta + \alpha\gamma\sigma_z\mathbb{E}[\epsilon]\beta\sigma_x\mathbb{E}_i[\xi_i] + \alpha(\gamma\sigma_z)^2\mathbb{E}[\epsilon^2] +$$
$$-\frac{1}{2}\left(\theta^2 + (\beta\sigma_x)^2\mathbb{E}_i[\xi_i^2]) + (\gamma\sigma_z)^2\mathbb{E}[\epsilon^2] + 2\theta\beta\sigma_x\mathbb{E}_i[\xi_i] + 2\theta\gamma\sigma_z\mathbb{E}[\epsilon] + \beta\sigma_x\mathbb{E}_i[\xi_i]\gamma\sigma_z\mathbb{E}[\epsilon]\right) di$$

.

Welfare

$$w(\theta) = \theta^2 + \alpha(\gamma\sigma_z)^2 \cdot 1 - \frac{1}{2}\left(\theta^2 + (\beta\sigma_x)^2 + (\gamma\sigma_z)^2\right);$$

$$w(\theta) = \frac{1}{2}\theta^2 - \frac{1}{2}(\beta\sigma_x)^2 - \frac{1}{2}(1 - 2\alpha)(\gamma\sigma_z)^2;$$

$$w(\theta) = \frac{1}{2}\theta^2 - \frac{1}{2}\Omega.$$

Welfare

Proposition 3 *Welfare necessarily increases with either an increase in δ or a reduction in σ*

Proof:

Ω can be easily rewritten as

$$\Omega = \frac{(1-2\alpha)+\alpha^2(1-\delta)}{[1-\alpha(1-\delta)]^2}\sigma^2;$$

$$\frac{\partial\Omega}{\partial\delta} < 0 \text{ with } \alpha \in \left[0, \frac{1}{2}\right);$$

$$\frac{-\alpha^2[1-\alpha(1-\delta)]^2 - 2[1-\alpha(1-\delta)]\alpha[(1-2\alpha)+\alpha^2(1-\delta)]}{[1-\alpha(1-\delta)]^4}\sigma^2 < 0;$$

$$-\alpha[1-\alpha(1-\delta)] - 2[(1-2\alpha)+\alpha^2(1-\delta)] < 0.$$

Welfare

$$\frac{\partial \Omega}{\partial \sigma} > 0 \text{ with } \alpha \in \left[0, \frac{1}{2}\right);$$

$$2\sigma \frac{(1-2\alpha) + \alpha^2(1-\delta)}{[1-\alpha(1-\delta)]^2} > 0.$$

Welfare

Proposition 4 (i) *A reduction in* σ_z *necessarily increases welfare.* **(ii)** *A reduction in* σ_x *decreases welfare if and only if* $\alpha > \frac{1}{3}$ and

$$\sigma_x^2 > \frac{(1-\alpha)^2}{3\alpha-1}\sigma_z^2.$$

Proof We can rewrite Ω in this way

$$\Omega = \frac{\sigma_x^2\sigma_z^2[(1-2\alpha)\sigma_x^2 + (1-\alpha)^2\sigma_z^2]}{[\sigma_x^2 + (1-\alpha)\sigma_z^2]^2};$$

$$\frac{\{2\sigma_z\sigma_x^2[(1-2\alpha)\sigma_x^2 + (1-\alpha)^2\sigma_z^2] + 2\sigma_x^2\sigma_z^3(1-\alpha)^2\}[\sigma_x^2 + (1-\alpha)\sigma_z^2]^2}{[\sigma_x^2 + (1-\alpha)\sigma_z^2]^4} + \\ - \frac{2[\sigma_x^2 + (1-\alpha)\sigma_z^2] \cdot 2(1-\alpha)\sigma_z\{\sigma_x^2\sigma_z^2[(1-2\alpha)\sigma_x^2 + (1-\alpha)^2\sigma_z^2]\}}{[\sigma_x^2 + (1-\alpha)\sigma_z^2]^4} \\ > 0.$$

Welfare

After a little bit of calculation

$$(1-2\alpha)\sigma_x^2 + (1-\alpha)\sigma_z^2 > 0.$$

True with $\alpha \in \left[0, \frac{1}{2}\right)$.

Proof (ii)

$$\frac{\partial \Omega}{\partial \sigma_x} < 0;$$

$$\frac{\{2\sigma_x\sigma_z^2[(1-2\alpha)\sigma_x^2 + (1-\alpha)^2\sigma_z^2] + 2\sigma_z^2\sigma_x^3(1-2\alpha)\}[\sigma_x^2 + (1-\alpha)\sigma_z^2]^2}{[\sigma_x^2 + (1-\alpha)\sigma_z^2]^4} +$$
$$-\frac{2[\sigma_x^2 + (1-\alpha)\sigma_z^2] \cdot 2\sigma_x\{\sigma_x^2\sigma_z^2[(1-2\alpha)\sigma_x^2 + (1-\alpha)^2\sigma_z^2]\}}{[\sigma_x^2 + (1-\alpha)\sigma_z^2]^4} < 0.$$

Welfare

$$\sigma_z^2(1-\alpha)^3 < [2(1-\alpha)^2 - (1-\alpha)^2 - 2(1-2\alpha)(1-\alpha)]\sigma_x^2;$$

$$\sigma_z^2(1-\alpha)^3 < [-1-3\alpha^2+4\alpha]\sigma_x^2;$$

$$\sigma_z^2(1-\alpha)^3 < (3\alpha-1)(1-\alpha)\sigma_x^2;$$

$$\sigma_x^2 > \frac{(1-\alpha)^2}{3\alpha-1}\sigma_z^2.$$

Strong Complementarities

In this section we see a "proper" global game. The game has the following structure:

$$\begin{cases} A = \theta \text{ if } K \leq r \\ A = \theta + 1 if\ K \geq r. \end{cases}$$

with $r \in [0, 1]$ For simplicity the author let $r = \frac{1}{2}$.

$$u(k_i, K) = Ak_i - k_i = k_i(A - 1) \text{ with } k_i \in [0, 1].$$

Suppose common knowledge, if $\underline{\theta} \equiv 0$ and $\bar{\theta} \equiv 1$. If $\theta \in [\underline{\theta}, \bar{\theta}]$, both $k_i = 1$ and $k_i = 0$ for all i would be an equilibrium.
Instead, with **heterogeneous** information, the possibility of multiple equilibria depends on the **transparency** of public information.

Equilibrium

An agent finds it optimal to invest $k_i = 1$ if $\mathbb{E}_i[A] \geq 1$. The authors restrict the attention to equilibria with monotonic strategies:

$$\forall\, x\ \exists\, x^*(z) : k_i = 1 \text{ if } x_i > x^*(z);$$
$$\forall\, x\ \exists\, x^*(z) : k_i = 0 \text{ if } x_i \leq x^*(z).$$

Thus, the aggregate investment is

$$K(\theta, z) = \Phi\left(\frac{\theta - x^*(z)}{\sigma_x}\right).$$

Equilibrium

This implies that

$$K \geq r \iff \theta \geq \theta^*(z);$$

where $\theta^*(z) = x^*(z)$.

$$\mathbb{E}[A|x_i, z] = (1-\delta)x_i + \delta z + \Phi\left(\frac{(1-\delta)x_i + \delta z - \theta^*}{\sigma}\right);$$

so we have $k_i = 1$ if $x \geq x^*$ and $k_i = 0$ otherwise, where x^* solves $\mathbb{E}[A|x^*, z] = 1$. Combining this condition with the previous result $\theta^* = x^*$ we have that the threshold must solve

$$F(x^*, z, \delta, \sigma) = (1-\delta)x^* + \delta z + \Phi\left(\frac{(1-\delta)x^* + \delta z - x^*}{\sigma}\right) = 1;$$

$$F(x^*, z, \delta, \sigma) = \delta(z - x^*) - \sigma\Phi^{-1}[1 - (1-\delta)x^* - \delta z] = 0.$$

Equilibrium

Let

$$\hat{\delta} \equiv \frac{\sqrt{2\pi}\sigma}{1+\sqrt{2\pi}\sigma};$$

and let $\hat{\sigma}_z$ be the unique **positive** solution to

$$\sigma_z^2\sqrt{2\pi}\sigma = \sigma_x^2\sqrt{\sigma_x^{-2}+\sigma_z^{-2}}. \quad (1)$$

We can compute $\hat{\sigma}_z$

$$\delta = \hat{\delta} \implies \frac{\sigma_x^2}{\sigma_x^2+\sigma_z^2} = \frac{\sqrt{2\pi}\sigma}{1+\sqrt{2\pi}\sigma};$$

$$\sigma_x^2 = \sqrt{2\pi}\sigma\sigma_z^2.$$

Substituting in to (1) we obtain

$$\sigma_z^2\sqrt{2\pi} = \sqrt{2\pi}\sigma\sigma_z^2\sqrt{(\sqrt{2\pi}\sigma_z^2)^{-1}+\sigma_z^2}. \quad (2)$$

Equilibrium

(2) has three solution, $\hat{\sigma}_z = 0; \hat{\sigma}_z \simeq -4.32$ and $\hat{\sigma}_z \simeq 2.32$. The only positive solution is the last. Then if $\sigma_z > 2.32$, F is monotonic in x for all z.
In the other case F is instead nonmonotonic, in this case for $z \in (\underline{z}, \bar{z})$ admits three possible solutions $x^*_{low} < x^*_{medium} < x^*_{large}$.
The two extreme solutions are stable equilibria, the intermediate one is an unstable equilibrium.

Welfare

The probability of coordination failure depends on the transparency of information.

$$Pr[z \in (\underline{z}, \bar{z})|\theta] = \Phi\left(\frac{\bar{z} - \theta}{\sigma_z}\right) - \Phi\left(\frac{\underline{z} - \theta}{\sigma_z}\right).$$

This conditional probability increases with $\bar{z}$, decreases with $\underline{z}$, and decreases with σ_z for $\theta \in (\underline{z}, \bar{z})$.

Conclusion

- This paper has examined the welfare effects of public and private information in an economy with investment complementarities.
- If the complementarity is weak there is an unique equilibrium, more transparency in public information increases welfare, despite the fact that it also increases volatility.
- If complementartities are strong, multiple equilibria are possible for high level of trasparency, more precise public information facilities more effective market coordination on either equilibrium.