Reading group on Global Games
a.a. 2021/2022
Session # 3

GLOBAL GAMES AND COMPLEMETARITY IN MACROECONOMICS & FINANCE

An Introduction

MILAN

February 2^{nd}, 2022

UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA

STRATEGIC COMPLEMENTARITY

in Macro & Finance

Other models with strategic complementarity

- beauty contests
- price setting with market power and multiple firms
- economic recessions
- real investment with spillovers and /or increasing returns

Continuum-player (CP) games with « threshold equilibria »

- rational bubbles in financial markets
- spillovers between real and financial investment

« Pure » global games and supermodular games

- currency attacks
- bank runs and financial fragility
- FDI and catalytic finance

Trend-chasing in
speculative behavior
→ Morris and Shin (1998)
→ Abreu and Brunnermeier (2003)

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→ Vives (1988, 2011, 2017)

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Lucas-Phelps
« Island economy »
→ Morris and Shin (2002)
→ Myatt and Wallace (2014)

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Dynamics of inequality and wealth distribution
→ Achdou et al. (2019)

MEAN-FIELD GAMES

- Other topics in macroeconomics that have been analyzed through the lens of strategic complementarity include:
  - ⟶ business cycles;
  - ⟶ endogenous growth;
  - ⟶ (sovereign) debt crises.

This presentation focuses on…

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- Two issues seem to limit the applicability of the global-games approach to macroeconomic modeling, as well as to question its validity as a realistic interpretational framework for macroeconomic analysis:

  - equilibrium market prices *aggregate* and *publicly disclose* traders' private information;

  - agents' aggregate behavior indirectly *reveals* – partially or completely – the private information that underpins it.

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posterior

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  } dispersion of conditional beliefs ↓ (posterior); "degree" of common knowledge ↑

- <u>**Recall**:</u> excessively precise public information is undesirable in global games for, in such environments, equilibrium uniqueness crucially hinges on **quasi-common-knowledge** induced by **private information**…

- Andrew Atkeson's "*Comment*" to Morris & Shin's "*Rethinking multiple equilibria in macroeconomic modeling*" [2000] openly underlines the problem…

> *« It is not clear to me how the argument presented by Morris and Shin would carry over to a model with markets. Their arguments require agents to have diverse beliefs about the probabilities of future outcomes in equilibrium, and this typically does not happen in models in which agents see the market signals about those probabilities embodied in asset prices. »*

p. 163

- The last section of the "*Comment*" epitomizes the critique…

> *« The question then stands, how do we integrate prices into the analysis and yet preserve the diversity of posterior beliefs across agents that is key to pinning down a unique equilibrium ? »*

p. 171

- The critique is compelling at two levels of analysis:

  - **first:** it questions the realism of the (key) assumption of quasi-common-knowledge. While in some strategic environments incomplete information is a realistic assumption, in the economic interactions macro models the opposite is true. In reason of the informational role of prices and/or other sources of endogenous learning, common knowledge is a more appropriate assumption;

  - **second:** it argues that, once endogenous information is included into the picture, the effectiveness of global games as a device for equilibrium selection vanishes – or, at least, it is significantly impaired.

- So…

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- So… **are global games doomed as a formal tool for macroeconomic analysis ?** Spoiler: NO!

HOW IS PUBLIC INFORMATION ENDOGENOUSLY CREATED ?

- Observation of Aggregates
- Observation of Market Prices

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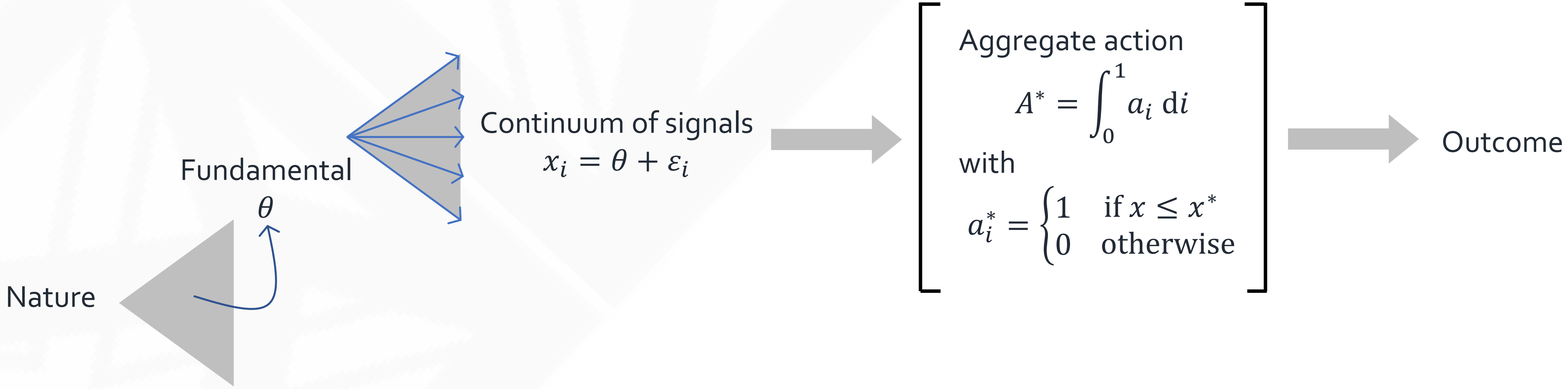
- **Observation of Aggregates**
- Observation of Market Prices

- Consider a generic, stylized global game played by a measure-one continuum of players $i \in [0,1]$ uniformly distributed over the unit interval;

- Its sequential structure can be summarized as follows…



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Nature

Fundamental
θ

Continuum of signals
$x_i = \theta + \varepsilon_i$

Aggregate action
$$A^* = \int_0^1 a_i \,\mathrm{d}i$$
with
$$a_i^* = \begin{cases} 1 & \text{if } x \leq x^* \\ 0 & \text{otherwise} \end{cases}$$

Outcome

Idiosyncratic
white noise
i.i.d. across agents

$$\begin{cases} \varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2), & \forall\, i \in [0,1] \\ \mathbb{E}[\varepsilon_i \varepsilon_j] = 0, & \forall\, i \neq j \end{cases}$$

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Outcome

Equilibrium unambiguously pinned down by the pair

$$(\,\theta, A^*(\theta)\,)$$

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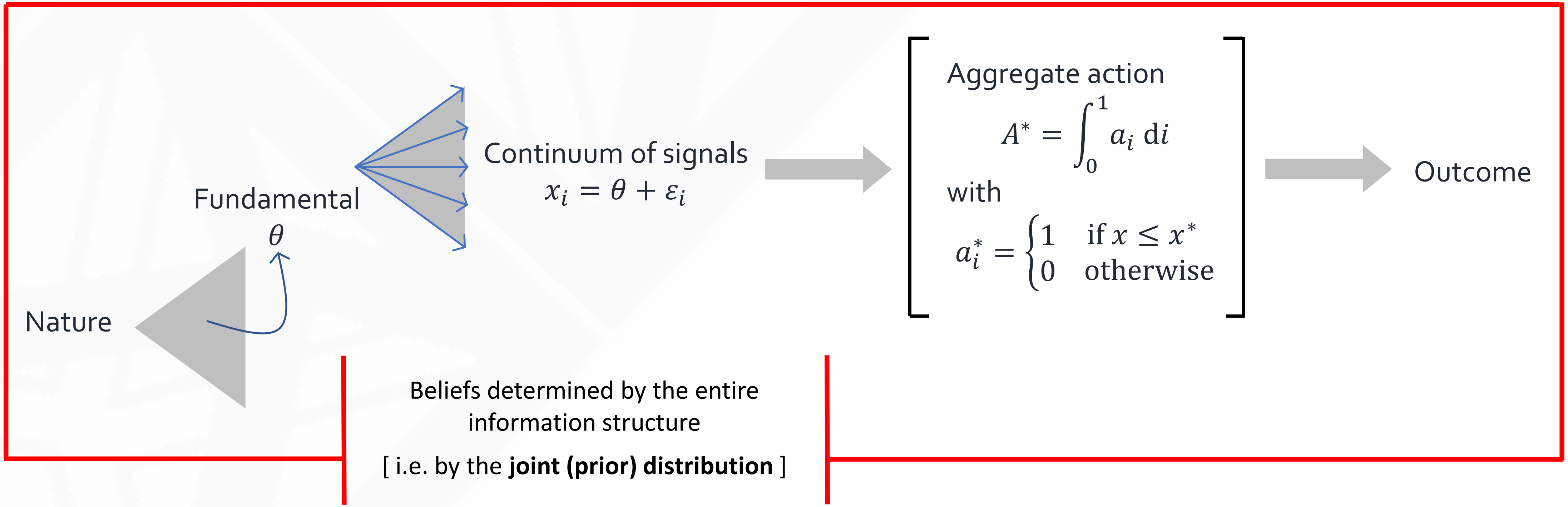

- In equilibrium, all players use monotone – threshold – strategies in the form

$$a_i^* = \begin{cases} 1, & \text{if } x_i \leq x^* \\ 0, & \text{if } x_i > x^* \end{cases}$$

- Since all idiosyncratic shoks ε_i are i.i.d. and all random variables are assumed to be independent, for any arbitrary realization $\theta = \tilde{\theta}$ of the fundamental, the *ex post* aggregate action A^* can be defined as

$$\begin{aligned} A^* &= \int_0^1 a_i \, \mathrm{d}i \\ &= Pr\left(x_i \leq x^* \mid \theta = \tilde{\theta} \right) \qquad \text{almost surely} \\ &= \Phi\left(\frac{x^* - \tilde{\theta}}{\sigma_\varepsilon}\right) \qquad \text{almost surely} \end{aligned}$$

…where $\Phi(\,\cdot\,)$ indicates the Normal Standard CDF.

- Note that

(A) In equilibrium, the (equilibrium) threshold x^* is common knowledge *by definition*.

(B) The information structure is common knowledge by assumption: as a consequence, the standard deviation σ_ε is common knowldge, too.

(C) The Normal Standard CDF $\Phi(\cdot)$ is strongly monone in its argument and invertible.

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IMPLICATION

SIGNAL EXTRACTION

$$A^* = \Phi\left(\frac{x^* - \tilde{\theta}}{\sigma_\varepsilon}\right) \longrightarrow \tilde{\theta} = x^* - \sigma_\varepsilon \Phi^{-1}(A^*)$$

Common knowledge is restored, and
so is equilibrium multiplicity

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A POSSIBLE SOLUTION: THE APPROACH OF DASGUPTA [2007]

Coordination and Delay in Global Games

- An emerging economy announces a liberalization program whereby Foreign Direct Investment (FDI) is encouraged

- Foreign investors can enter the program early (in $t = 1$) or late (in $t = 2$)

- FDIs entail a fixed cost $c > 0$, and yield stochastic returns jointly determined by:

    - the (unknown) fundamentals of the economy, summarized by the unidimensional statistic $\theta \in \mathbb{R}$;

    - the total amount $A \in [0,1]$ of resources invested – with $A = A_1 + A_2$.

- Formally, the returns on FDIs are defined as follows:

$$R(\theta) = \begin{cases} b_t > c & \text{if } A \geq 1 - \theta \\ 0 & \text{if } A < 1 - \theta \end{cases}$$

with $b_1 > b_2$.

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Delay is inefficient

$$R(\theta) = \begin{cases} b_t > c & \text{if } A \geq 1 - \theta \\ 0 & \text{if } A < 1 - \theta \end{cases}$$

with $b_1 > b_2$.

- All investors share an improper (uninformative) common prior for the unobserved economic fundamental θ, i.e.

$$\theta \sim \mathcal{U}(\mathbb{R})$$

- Before any FDI occurs, all prospective investors observe a private signal about θ, in the form

$$x_1^i = \theta + \varepsilon_1^i$$

- Since all FDIs yield positive returns only if the total investment exceeds a critical mass $(1-\theta)$, early investors' monotone strategies played in equilibrium at date $t = 1$ are in the form

$$a_1^*(x_1^i) = \begin{cases} 1, & \text{if } x_1^i \geq x_1^* \\ 0, & \text{if } x_1^i < x_1^* \end{cases}$$

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To opt for early participation, investors must be sufficiently optimistic about θ

- Delay is costly, but entails a significant **informational advantage**:

  - late investors are allowed to *privately* observe the mass of early investors before deciding whether or not to enter the program;

  - formally, such private information comes in the form of a noisy signal defined as follows

$$x_2^i = \Phi^{-1}(A_1) + \varepsilon_2^i$$

...where A_1 is the mass of early investors, and $\Phi^{-1}(\cdot) : (0,1) \longmapsto \mathbb{R}$ indicates the inverse function of the Normal Standard CDF.

- Recall that, in the presence of a continuum of i.i.d. random shocks, we have that

$$A_1^* = \Phi\left(\frac{\theta - x_1^*}{\sigma_\varepsilon}\right) \quad \text{almost surely}$$

- We can therefore rewrite the additional private information available to late investors as

$$x_2^i = \Phi^{-1}(A_1) + \varepsilon_2^i$$

$$= \Phi^{-1}\left(\Phi\left(\frac{\theta - x_1^*}{\sigma_\varepsilon}\right)\right) + \varepsilon_2^i$$

$$= \frac{\theta - x_1^*}{\sigma_\varepsilon} + \varepsilon_2^i$$

...so that trivial signal extraction yelds

$$\sigma_\varepsilon\, x_2^i - x_1^* \;=\; \theta + \sigma_\varepsilon\, \varepsilon_2^i$$

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Observed

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Common knowledge in equilibrium

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RANDOM!

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...so that trivial signal extraction yelds

$$\underbrace{\sigma_\varepsilon\, x_2^i - x_1^*}_{\text{NON RANDOM}} \;=\; \underbrace{\theta + \sigma_\varepsilon\, \varepsilon_2^i}_{\text{RANDOM}}$$

NON RANDOM = RANDOM

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$$= \Phi^{-1}\left(\Phi\left(\frac{\theta - x_1^*}{\sigma_\varepsilon}\right)\right) + \varepsilon_2^i$$

$$= \frac{\theta - x_1^*}{\sigma_\varepsilon} + \varepsilon_2^i$$

...so that trivial signal extraction yelds

$$z_i = \theta + k\, \varepsilon_2^i$$

- We can therefore rewrite the additional private information available to late investors as

$$x_2^i = \Phi^{-1}(A_1) + \varepsilon_2^i$$

$$= \Phi^{-1}\left(\Phi\left(\frac{\theta - x_1^*}{\sigma_\varepsilon}\right)\right) + \varepsilon_2^i$$

$$= \frac{\theta - x_1^*}{\sigma_\varepsilon} + \varepsilon_2^i$$

...so that trivial signal extraction yelds

$$z_i = \theta + k\,\varepsilon_2^i$$

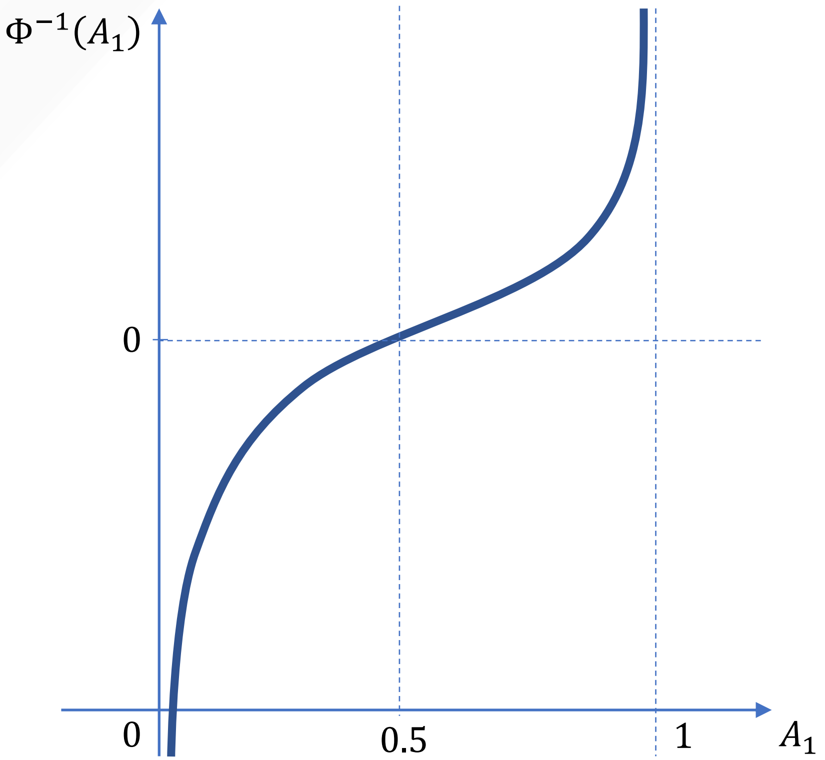
Endogenous private info
Is still noisy!

$$z_i \mid \theta \sim \mathcal{N}(\theta, k^2\sigma_\varepsilon^2)$$

with

$$k = \sigma_\varepsilon$$

➢ **CAVEAT LECTOR:** the function $\Phi^{-1}(\cdot)$ is – highly – nonlinear!

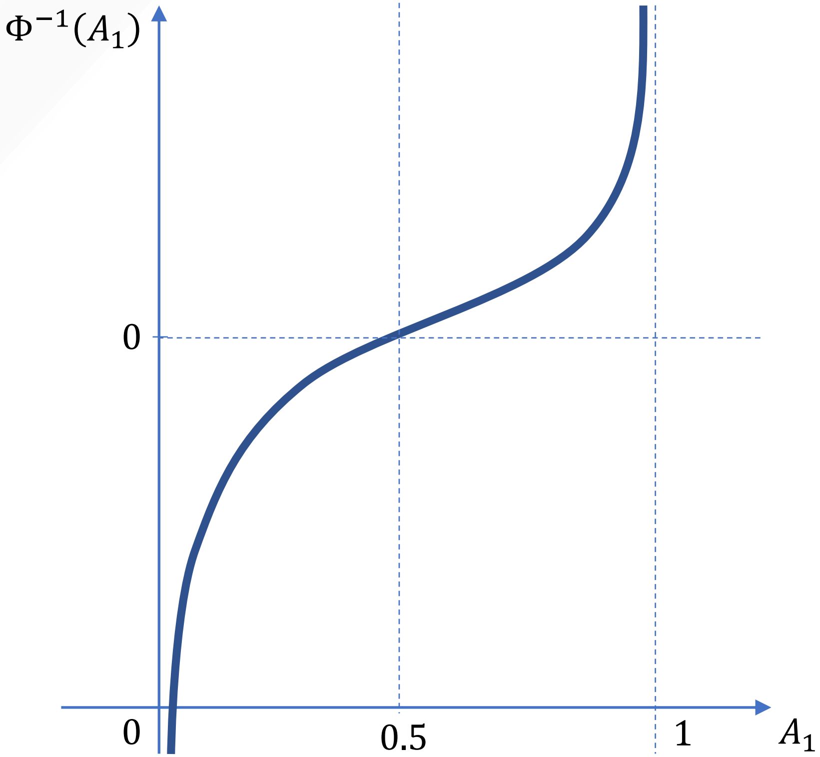


➢ **CAVEAT LECTOR:** the function $\Phi^{-1}(\cdot)$ is – highly – nonlinear!

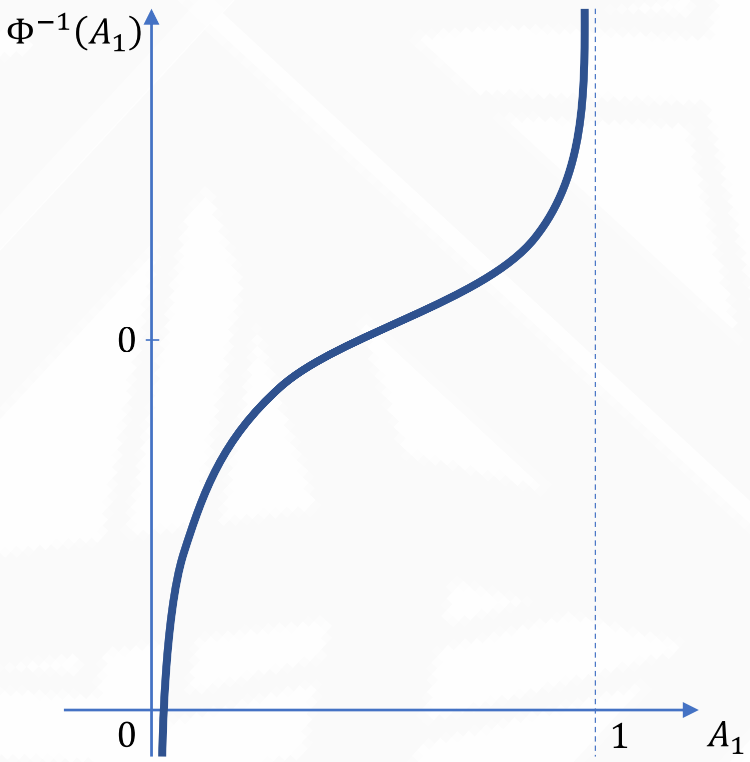
➢ As a consequence, the specification of the signal-generating technology that governs late investors' private information is **not without loss of generality**…

… as the author underlines [see Footnote 7, p. 201].

➢ A graphical example may help clarify the issue…



➢ Graphically, the intuition is the following…



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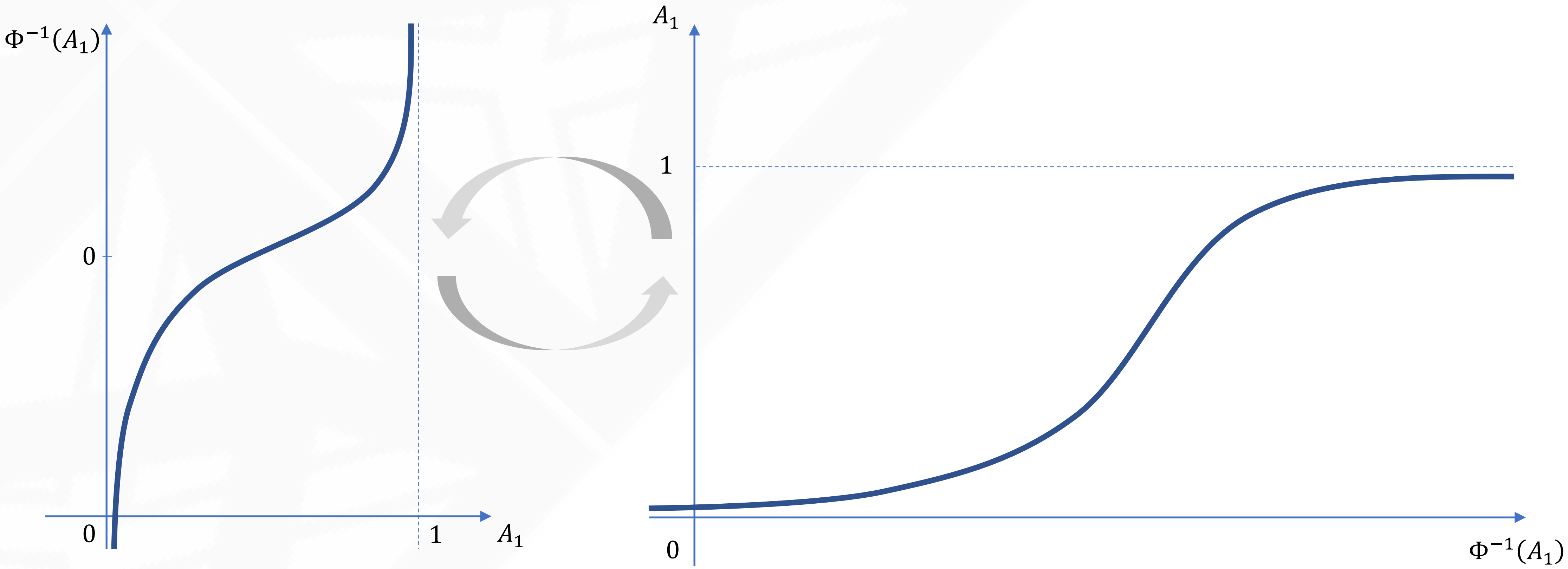

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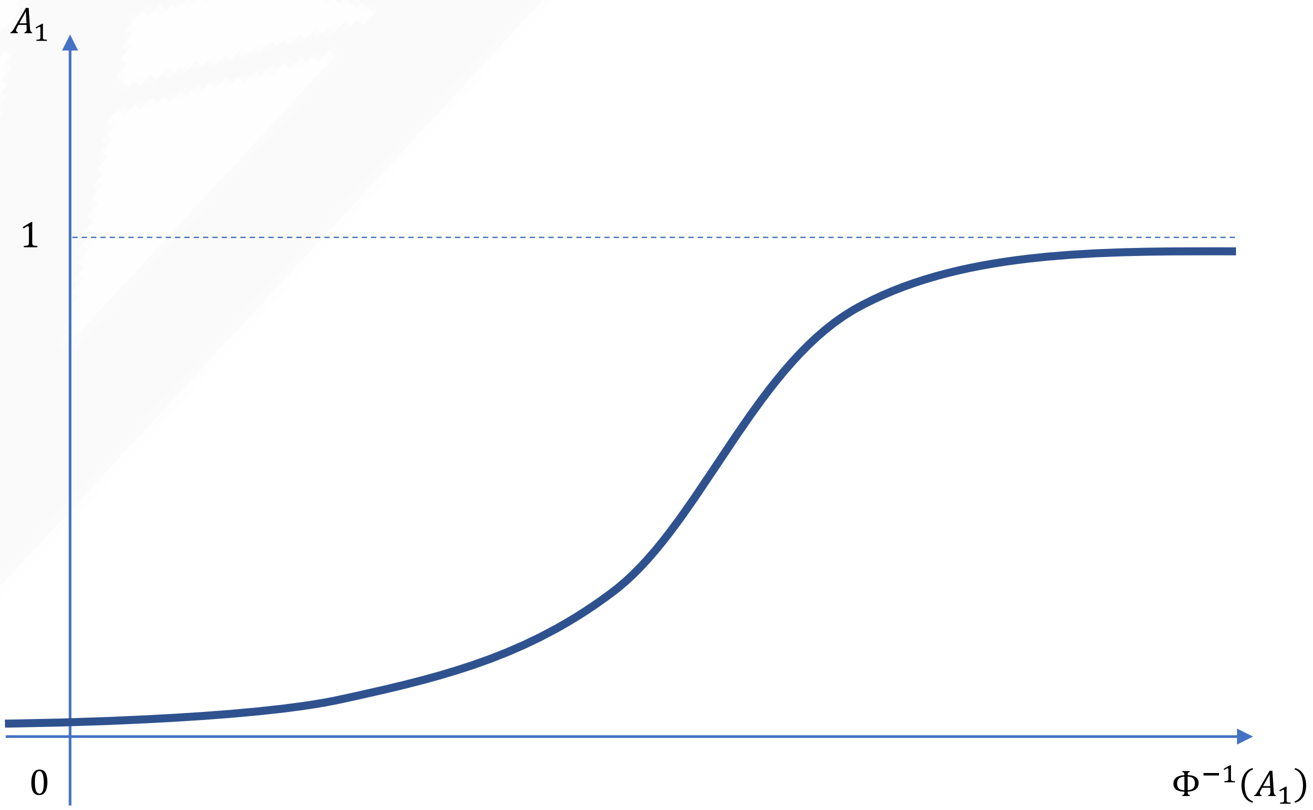
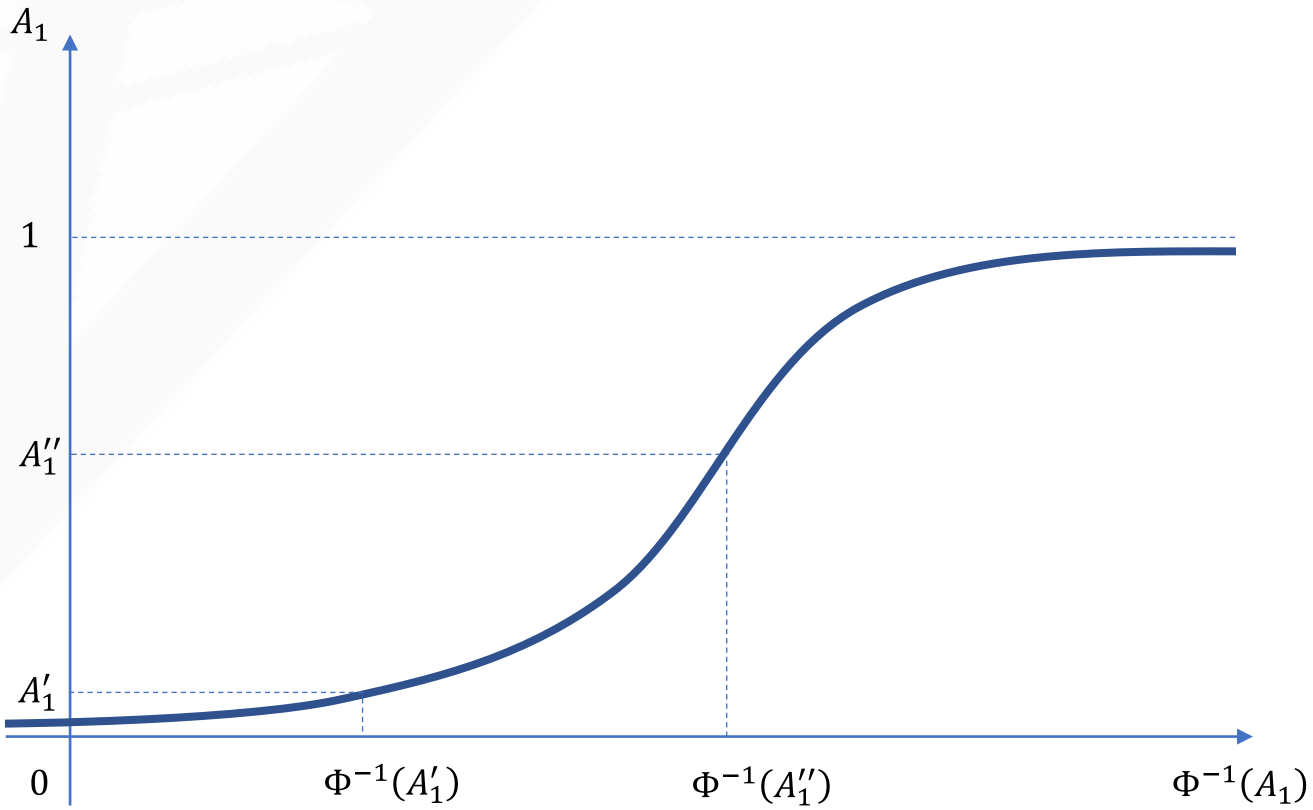


➢ Graphically, the intuition is the following…

❑ Fix two arbitrary levels of early investment A_1' and A_1'', with

$$A_1'' > A_1'$$

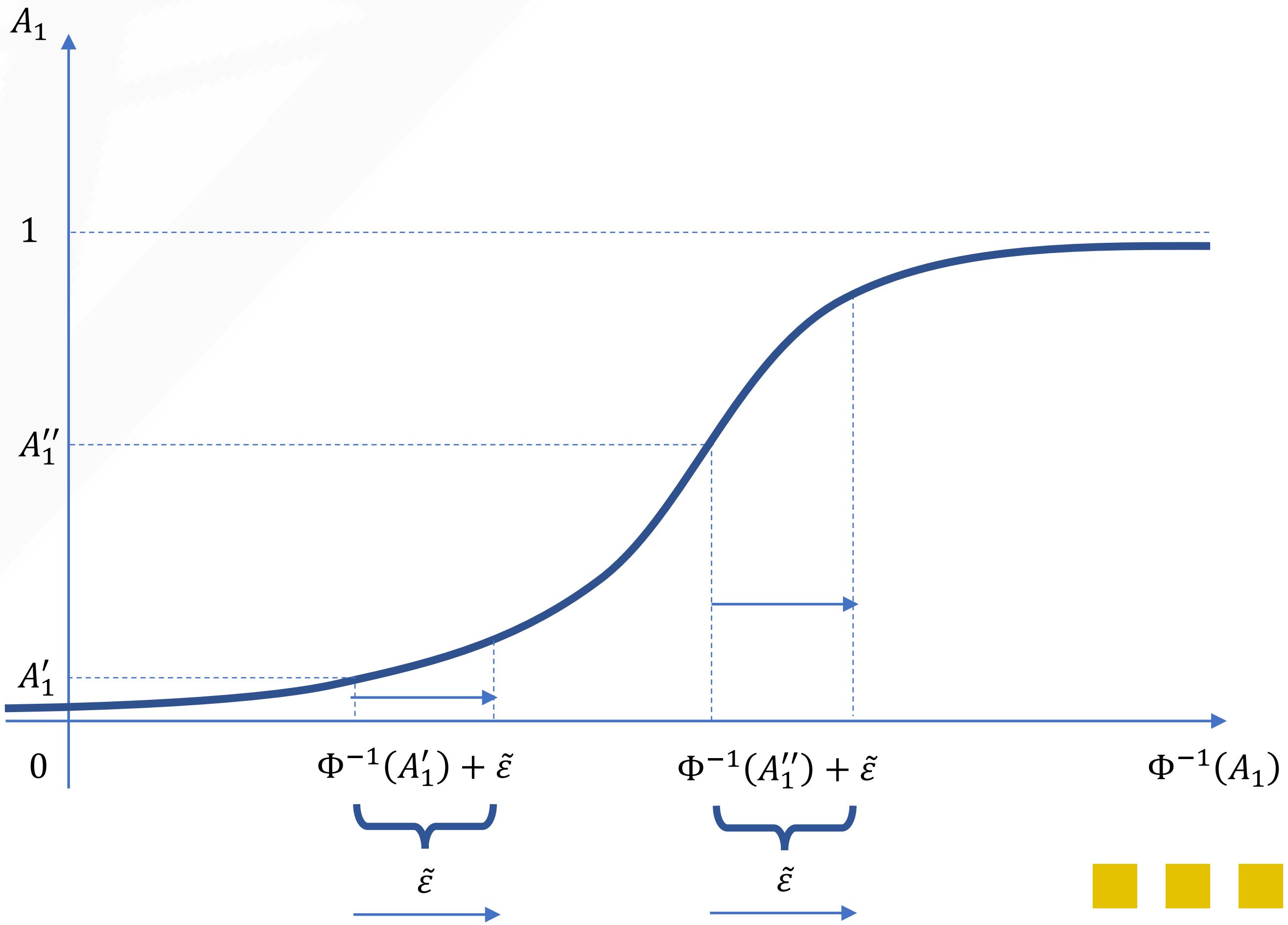


- Graphically, the intuition is the following…

  - Fix two arbitrary levels of early investment A_1' and A_1'', with

$$A_1'' > A_1'$$

  - Transform the input of the signal via $\Phi^{-1}(\cdot)$ and apply the same arbitrary shock $\varepsilon = \tilde{\varepsilon}$ …



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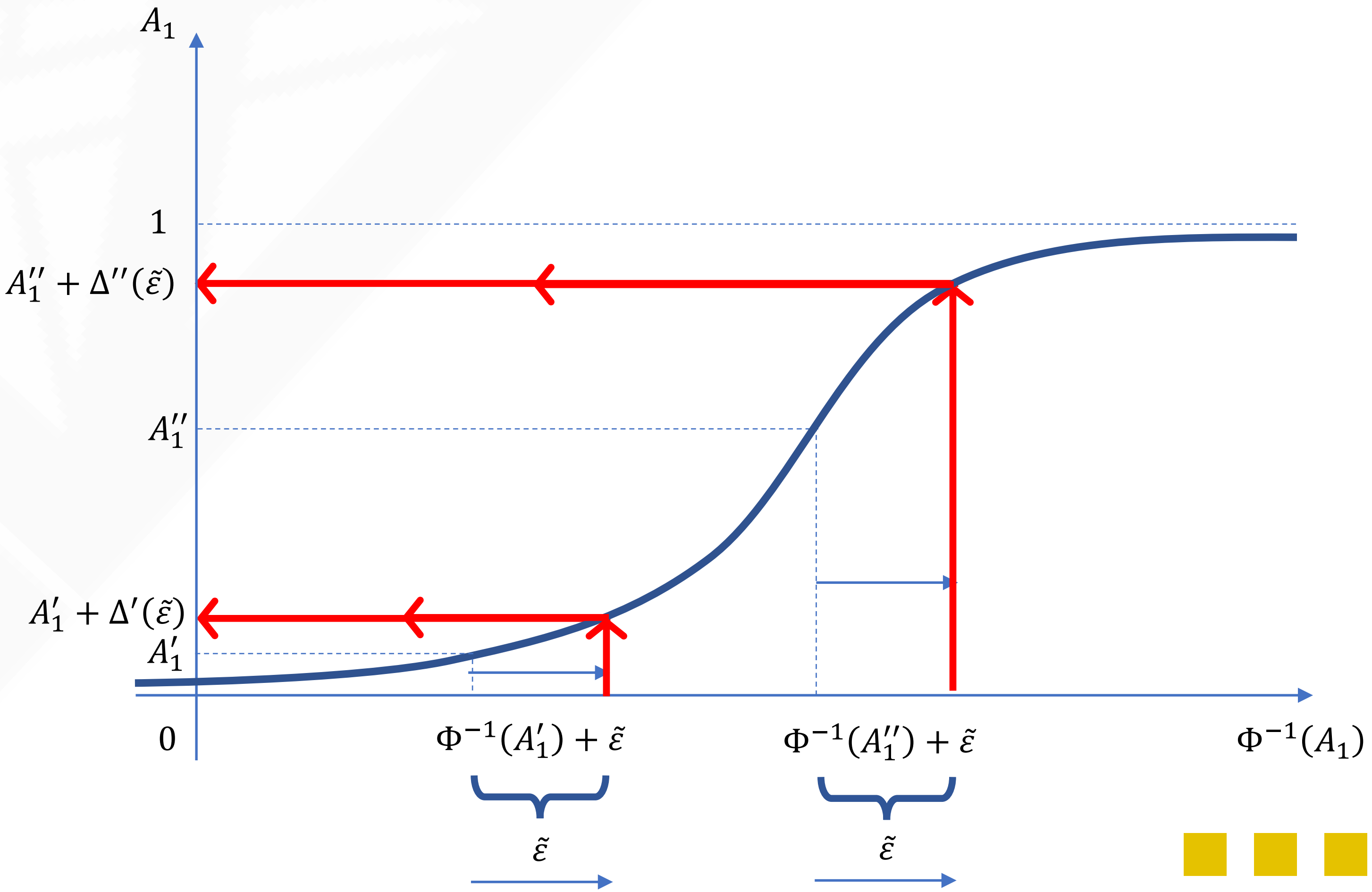
- ❑ Transform the input of the signal via $\Phi^{-1}(\cdot)$ and apply the same arbitrary shock $\varepsilon = \tilde{\varepsilon}$, and …

- ❑ … map the result back into the domain $[0,1]$ of A.



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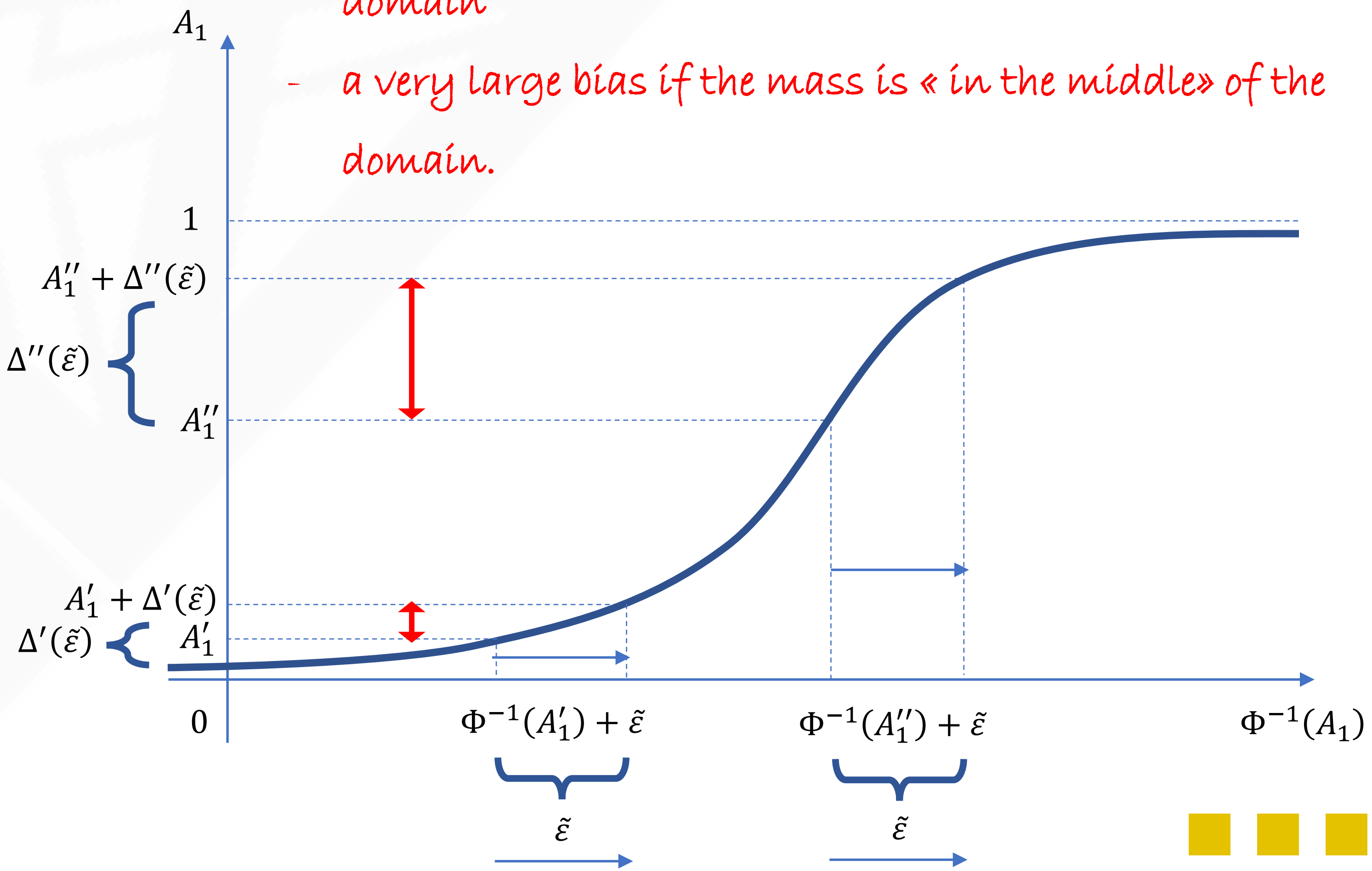
- … map the result back into the domain $[0,1]$ of A.

The same shock into the signal space, in the original domain of masses amounts to

- a small bias if the mass is at the « etremes » of the domain
- a very large bias if the mass is « in the middle» of the domain.



HOW IS PUBLIC INFORMATION ENDOGENOUSLY CREATED ?

- Observation of Aggregates
- Observation of Market Prices

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- Observation of Aggregates
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- Improper uniform priors with normally distributed noise are convenient to improve the tractability of models;

- however, improper priors prevent a well-defined specification of conditioanl correlation structures, that are of key importance to the analysis of models with endogenous information;

- virtually all models with learning-from-prices fall into the category, hence **proper (informative) priors** must be used.

- Consider a vector $\vec{\omega}$ of random variables of interest (the **fundamentals**), and a second random vector $\vec{s}$ of **signals**, that summarize the information available about the fundamentals to a rational (Bayesian) decision maker;

- if the joint prior distribution is a multivariate Normal, i.e.

$$\begin{bmatrix} \vec{\omega} \\ \cdots \\ \vec{s} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_\omega \\ \cdots \\ \mu_s \end{bmatrix}, \begin{bmatrix} \Sigma_{11} & \vdots & \Sigma_{12} \\ \cdots & \cdots & \cdots \\ \Sigma_{21} & \vdots & \Sigma_{22} \end{bmatrix} \right)$$

then the posterior distribution of the fundamentals conditional to the signals is a multivariate Normal with mean $\bar{\mu}$ variance/covariance $\bar{\Sigma}$ defined as follows

$$\bar{\mu} = \mu_\omega + \Sigma_{12}\, \Sigma_{22}^{-1}\, (\vec{s} - \mu_s)$$

$$\bar{\Sigma} = \Sigma_{11} - \Sigma_{12}\, \Sigma_{22}^{-1}\, \Sigma_{21}$$

LEARNING FROM PRICES

A Toy Model

- In Walrasian markets with infinitely many atomistic agents, clearing prices aggregate information more or less in the same way the observation of aggregate actions (e.g. investment and/or trading volumes) does

  - the focus on monotone equilibrium strategies, coupled with – more or less warranted – extensions of LLN arguments to continua of i.i.d. random variables, allow to establish that **true conditional probabilities equal actual ex post masses almost surely**;

  - once aggregated in the form of a quantity or a schedule, the private information possessed by agents "flows" into market prices via market clearing;

  - the intuition is relatively simple: every mass is matched to its market counterparty by a **unique clearing price**, hence it is always possible to perform "reverse engineering" onto such price to (partially) recover the demand/supply that generated it – hence, the information that the latter contains.

- Consider a stylized economy that lasts for three periods $t \in \{0,1,2\}$ with a single financial asset, where…
  - … a continuum of atomistic agents, indexed by i, possess the entire mass of assets at the initial date $t = 0$;
    - one agent ⟶ one asset ;
  - a second continuum of atomistic agents, indexed by j, possesses only money, and is willing to buy the asset if the price is sufficiently low ;
    - one agent ⟶ one asset, once again…
  - both continua are of unitary mass, with agents uniformly distributed over the interval $[0,1]$;
  - the asset pays an unknown (log) amount of money $\theta \in \mathbb{R}$ at date $t = 2$, and can be traded at the interim date $t = 1$ into a competitive financial market.

- Prior to trade, all traders observe private signals in the form

$$x_i = \theta + k\,\eta + \varepsilon_i$$

$$x_j = \theta + k\,\eta + \varepsilon_j$$

with $k \geq 0$ an arbitrary scalar, and with

$$\eta \sim \mathcal{N}(0, \sigma_\eta^2)$$

$$\varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

and where all random variables are assumed to be independent.

- Consider the following generic monotone strategies

$$sell \iff x_i - \alpha_I\, p \;\le\; \beta_I \Longrightarrow p \;\ge\; f_I(x_i)$$

$$buy \iff x_j - \alpha_J\, p \;\ge \beta_J \Longrightarrow p \;\le\; f_J(x_J)$$

that in turn entail that the aggregate market supply and demand for assets, respectively, can be expressed as conditional probabilities in the form

$$S(p; f_I\,) = Pr(\;\varepsilon_i \le \beta_I + \alpha_I\, p - (\theta + \eta)\;\mid\; \theta, \eta\,)$$

$$D\big(p; f_J\,\big) = Pr\big(\;\varepsilon_j \ge\; \beta_J + \alpha_J\, p - (\theta + \eta)\;\mid\; \theta, \eta\,\big)$$

- All coefficients α and β are to be determined in equilibrium.

➢ Aggregate demand and supply can therefore be expressed as

$$S(p;\alpha_I) = \Phi\left(\frac{\beta_I + \alpha_I p - (\theta + k\,\eta)}{\sigma_\varepsilon}\right)$$

$$D(p;\alpha_J) = \Phi\left(\frac{(\theta + k\,\eta) - \beta_J - \alpha_J p}{\sigma_\varepsilon}\right)$$

so that market clearing yields

$$\Phi\left(\frac{\beta_I + \alpha_I p - (\theta + k\,\eta)}{\sigma_\varepsilon}\right) = \Phi\left(\frac{(\theta + k\,\eta) - \beta_J - \alpha_J p}{\sigma_\varepsilon}\right)$$

so that, solving in the (log) clearing price, we obtain

$$p^*(\theta,\eta) = \left(\frac{1}{\alpha_I + \alpha_J}\right)\left(-(\beta_I + \beta_J) + 2\,\theta + 2k\,\eta\,\right)$$

- Being a linear function of θ and η, the equilibrium clearing price is an implicit informative signal;

- Since the information structure is common knowledge by assumption, and all endogenous coefficients are common knowledge in equilibrium, upon observing p^* every agent is able to perform the following signal extraction:

$$p^* = \left(\frac{1}{\alpha_I + \alpha_J}\right)\left(-(\beta_I + \beta_J) + 2\,\theta + 2k\,\eta\right)$$

$$\underbrace{\frac{(\alpha_I + \alpha_J)\,p^* + (\beta_I + \beta_J)}{2}} = \underbrace{\theta + k\,\eta}$$

NON RANDOM = RANDOM

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- Rational agents shoul recognize *ex ante* the informational role of $\tilde{p}^*$, hence their equilibrium monotone strategies **must** be consistent with their expected capital gain from trade given model-consistent expectations, i.e.

$$x_i - \alpha_I\, p \;\leq\; \beta_I \quad \Longrightarrow \quad \mathbb{E}[\theta | x_i, \tilde{p}^*] \leq p^* \quad \Longrightarrow \quad sell$$

$$x_j - \alpha_J\, p \;\geq \beta_J \quad \Longrightarrow \quad \mathbb{E}[\theta | x_i, \tilde{p}^*] \geq p^* \quad \Longrightarrow \quad buy$$

- **PROBLEM!** Compare the exogenous private signals availabe to agents, i.e.

$$x_i = \theta + k\,\eta + \varepsilon_i$$

$$x_j = \theta + k\,\eta + \varepsilon_j$$

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$$k = 0$$

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The equilibrium price reveals the

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GROSSMAN-STIGLITZ PARADOX

GLOBAL GAMES
AND COMPLEMENTARITY
IN MACROECONOMICS
& FINANCE

An Introduction

MILAN

February 2nd, 2022

Thank You!

UNIVERSITA' DEGLI STUDI DI MILANO BICOCCA