Regime Change and Equilibrium Multiplicity

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Study Phenomenon

Models of mass uprisings: Models that study the possibility that within a population some citizens may rebel against the current system with the goal of regime change

Literature

1. **"Coordination problem/Collective action"** approach
   - Schelling (1960)
   - Weingast (1997)
2. **"Global games of regime change"** approach
   - Edmond, 2013
   - Egorov, Guriev and Sonin, 2009
   - Persson and Tabellini, 2009
   - Little, 2012
   - Boix and Svolik, 2013

Instrument - Regime Change Game

Instrument chosen by the author: **Regime Change Game**
Features: **Incomplete Information Coordination Game**

1. Players may be uncertain about:
    - How strong the regime is
    - The preferences of other players
    - Variety of other factors
2. Each player chooses whether to attack a regime and the regime falls if and only if enough players attack

Why not complete information game?

The **complete information** typically outlines a setting in which a regime falls if and only if enough people mobilize and in which people want to mobilize if and only if the regime will in fact fall.

In such an environment there are **two pure strategy equilibria**:

1. one in which **no citizens mobilize**
2. one in which **all the citizens mobilizes**

In such models, changes from one equilibrium to another are discontinuous, a structural or strategic intervention (by the government, opposition, media,..) either does not matter at all, or it radically shifts the course of events.

This fact limits the scope of phenomena that can potentially be explained with complete information coordination models

Advantage of incomplete information game

The presence of **incomplete information** typically implies that there are pure strategy equilibria in which players use **cutpoint strategies**.
A player participates if her private information is favorable enough and does not participate otherwise.

In this context the players behaviour can change continuously in response to local changes in structural environment or strategical intervention by other actors causing change in the cutpoint thereby inducing changes in the level of mobilization or the probability of the regime falling (**smoothness equilibrium behaviour**).

It is possible to use these models to study how the risk of mass uprisings relates to a host of important strategic and structural factors

Research Question 1

Schelling (1960) and Weingast (1997) have argued that mass uprisings are a coordination problem which rely on equilibrium multiplicity.

Global games of regime change incorporate much of the structure of coordination games, but typically yield a unique equilibrium and they have the desirable feature of smoothness equilibrium behaviour.

Is equilibrium uniqueness in fact a natural and robust feature of incomplete information models of mass uprisings?

Research Question 2

If not, is it possible to retain the desirable smoothness properties associated with global games of regime change while avoiding unrealistic assumptions that drive equilibrium uniqueness?

Paper's Idea

Combine literature findings with desirable properties of global games in a consistent manner

Starting point

A regime change game becomes a global game, and thus has a unique equilibrium if it satisfies:

1. **Two-sided limit dominance**: players' beliefs assign positive probability to a state of the world in which it is a dominant strategy to attack the regime and to a state of the world in which it is a dominant strategy not to attack the regime
2. **Thick tails**: probability assigned to those states by the common prior is sufficiently large

The Game

		$N < T$	$N \geq T$
Player i	$a_i = 0$	0	0
	$a_i = 1$	$-k$	$\theta - k$

Canonical regime change game developed by Morris and Shin (2004)[1]

- continuum of individuals (of mass 1), each of whom makes binary choice $a_i \in \{0, 1\}$
- regime change is achieved if $N \geq T$, where N is the mass of participants
- k is the cost to participate
- T is measure of regime strength
- θ is measure of the quality of potential replacement regimes

[1] Angeletos, Hellwig and Pavan (2006,2007)

Double Scenario

To examine the two research questions the author studies a canonical regime change game under **two different assumptions on the informational environment**:

1. players are **uncertain of the payoffs** from overturning the regime
2. players are **uncertain of the regime's strength**

Scenario 1 - Payoff Uncertainty

Suppose there is uncertainty over the payoff from regime change θ

1. If $\theta < k$ *not to participate* is dominant strategy
2. *participate* is never a dominant strategy

This scenario induces **one-side limit dominance** because varying θ only $\{a_i = 0\}$ can be dominant strategy

Scenario 2 - Threshold Uncertainty

Suppose there is uncertainty over the threshold T and the players' beliefs assign positive probability to $\{T < 0\}$ and $\{T > 1\}$

1. If $\theta < k$ *not to participate* is dominant strategy
2. If $\theta > k$ and $\{T > 1\}$ *not to participate* is dominant strategy
3. If $\theta > k$ and $\{T < 0\}$ *participate* is dominant strategy

In this scenario formally it is possible to observe **two-side limit dominance** because varying T both the actions can be dominant strategy

Scenario 2 - $T < 0$

$T < 0$: citizens believe it is possible that the regime will fall even if no one participates in a mass uprising

This is **not an unreasonable assumption** because regimes may fall for all manner of reasons without a rebellion

Criticality: in the event that the regime is so weak that it will fall regardless of the presence of a rebellion, if a single measure-zero person turns out to protest, she derives the extra benefit θ from having participated in the rebellion that led to the regime falling, even though she was in fact irrelevant to the outcome.

!: Condition III seems hard to motivate, but it will be crucial for equilibrium uniqueness

A step over... Scenarios' Equilibria

The author characterizes the equilibrium correspondence for the regime change game under each form of uncertainty

Payoff Uncertainty

Each player receives a **signal** $s_i = \theta + \epsilon_i$ where:

1. θ is drawn from $N(m, \sigma_\theta^2)$
2. $\epsilon_i \sim N(0, \sigma_\epsilon^2)$
3. θ, ϵ_i indipendent

Define:

1. R as the set of possible combinations of the game's parameters $(m, k, T, \sigma_\epsilon, \sigma_\theta)$
2. Γ^θ as the game
3. A specific instance of the game with parameter value $r \in R$ is $\Gamma^\theta(r)$

The posterior beliefs about θ are:

$$\theta|\theta_i \sim N(\overline{m}_i = \lambda s_i + (1-\lambda)m, \quad \sigma_\lambda^2 = \lambda\sigma_\epsilon^2) \quad \lambda = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2}$$

Payoff Uncertainty

Equilibrium is derived through **Bayesian equilibria in cutoff strategies**

All players adopt the same strategy and that strategy takes the form:

$$\{a_i = 1\} \iff s_i \geq \hat{s}$$

Where $\hat{s}$ is the **equilibrium cutoff** (which may be finite or infinite)

Payoff Uncertainty

Suppose player i believes all players j participate if and only if:

$$s_j \geq \hat{s} \iff \epsilon_j \geq \hat{s} - \theta$$

Then, for a given θ, player i anticipates total participation:

$$P(\epsilon_j \geq \hat{s} - \theta) = 1 - \Phi\left(\frac{\hat{s} - \theta}{\sigma_\epsilon}\right)$$

Hence, player i believes regime change will be achieved if and only if $\theta \geq \theta^*(\hat{s}, r)$ with :

$$1 - \Phi\left(\frac{\hat{s} - \theta^*(\hat{s}, r)}{\sigma_\epsilon}\right) = T \iff \theta^*(\hat{s}, r) = \hat{s} - \Phi^{-1}(1 - T)\sigma_\epsilon$$

Payoff Uncertainty

From the perspective of a player i the probability of regime change becomes:

$$P(\theta \geq \theta^*(\hat{s}, r)) = 1 - \Phi\left(\frac{\theta^*(\hat{s}, r) - \overline{m}_i}{\sigma_\lambda}\right)$$

Hence a player will participate if the utility is greater than k:

$$\left[1 - \Phi\left(\frac{\theta^*(\hat{s}, r) - \overline{m}_i}{\sigma_\lambda}\right)\right] \mathbb{E}[\theta | \theta \geq \theta^*(\hat{s}, r), s_i] \geq k$$

$$\mathbb{E}[\theta | \theta \geq \theta^*(\hat{s}, r), s_i] = \overline{m}_i + \sigma_\lambda \frac{\phi\left(\frac{\theta^*(\hat{s}, r) - \overline{m}_i}{\sigma_\lambda}\right)}{1 - \Phi\left(\frac{\theta^*(\hat{s}, r) - \overline{m}_i}{\sigma_\lambda}\right)}$$

Payoff Uncertainty

Notation:

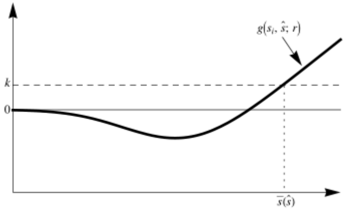
$$g(s_i, \hat{s}, r) \equiv \left[1 - \Phi\left(\frac{\theta^*(\hat{s}, r) - \overline{m}_i}{\sigma_\lambda}\right)\right] \mathbb{E}[\theta|\theta \geq \theta^*(\hat{s}, r), s_i]$$

It is necessary to verify the compatibility of the function with the existence of the cutoff rule

Payoff Uncertainty - Description of $g(s_i, \hat{s}, r)$

Lemma: The following result establishes sufficient conditions for player i using a cutoff rule given that all others do:

1. $\lim_{s_i \to -\infty} g(s_i, \hat{s}, r) = 0$
2. $\lim_{s_i \to \infty} g(s_i, \hat{s}, r) = \infty$
3. There is
   - exactly one $\bar{s}(\hat{s})$ satisfying $g(\bar{s}(\hat{s}), \hat{s}, r) = k$
   - for all $s_i < \bar{s}(\hat{s})$, $g(s_i, \hat{s}, r) < k$
   - for all $s_i > \bar{s}(\hat{s})$, $g(s_i, \hat{s}, r) > k$



Payoff Uncertainty - Lemma's criticality 1

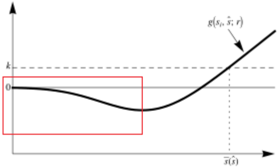
It is not specified whether the three properties are satisfied for a fixed $\hat{s}$

Payoff Uncertainty - Lemma's criticality 2

We consider the trend of the function described by the drawing to be correct.

The function becomes monotonically increasing in s_i before being positive

However for the group of citizens with $s_i < \bar{s}(\hat{s})$ there is a portion for which as anti-government sentiment increases, the utility decreases.



Payoff Uncertainty

An **equilibrium cutoff** must satisfy the condition:

$$g(\hat{s}, \hat{s}, r) = k$$

That is, a player whose signal is right at the cutoff ($s_i = \hat{s}$) must be indifferent between participating and not

Notation:

$$G^\theta(\hat{s}, r) \equiv g(\hat{s}, \hat{s}, r)$$

It is necessary to characterise the number of possible equilibrium cutoffs

Payoff Uncertainty

Lemma: A finite cutoff rule $\hat{s}$ is a finite cutoff equilibrium of $\Gamma^\theta(r)$ if and only if it satisfies

$$G^\theta(\hat{s}, r) = k$$

Here we have the condition related to the **level of the function**

Lemma: $\forall r \in R$, $\Gamma^\theta(r)$ has the following properties:

1. $\lim_{\hat{s} \to -\infty} G^\theta(\hat{s}, r) = -\infty$
2. $\lim_{\hat{s} \to \infty} G^\theta(\hat{s}, r) = 0$
3. $G^\theta(\hat{s}, r)$ has a single peak

Here we have the **shape of the function**

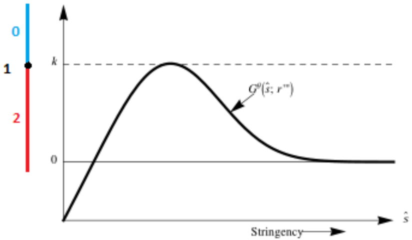
Payoff Uncertainty

Lemma: Consider r_{-k} the parameter r without the component k. Let R^*_{-k} be the set of parameter values satisfying the following: For any (r_{-k}, k) with $r_{-k} \in R^*_{-k}$, $\arg\max_{\hat{s}} G^{\theta}(\hat{s}, r) \geq 0$

1. for any $r \in R$ the game $\Gamma^{\theta}(r)$ has an infinite cutoff equilibrium
2. for any r_{-k} there is an open set $O_{-}(r_{-k})$such that for $k \in O_{-}(r_{-k})$ the game $\Gamma^{\theta}(r_{-k}, k)$ has no finite cutoff equilibria
3. for any $r_{-k} \in R^*_{-k}$
    - there is an open set $O_{+}(r_{-k})$such that for $k \in O_{+}(r_{-k})$ the game $\Gamma^{\theta}(r_{-k}, k)$ has two finite cutoff equilibria
    - There is exactly one k such that the game $\Gamma^{\theta}(r_{-k}, k)$ has one finite cutoff equilibria

The characterisation of the equilibrium cutoff can be summarised by the following figure

Equilibria according to the author



Given m, $T, \sigma_\epsilon, \sigma_\theta$ and the function

1. There is a single value of k for which there is one finite $\hat{s}$
2. There is an open set of k values for which there are two finite $\hat{s}$
3. There is an open set of k values for which there is no finite $\hat{s}$
4. There is always an infinite $\hat{s}$

Equilibria according to the author

The author states that the game Γ^θ has either one or three cutoff equilibria

1. For values of r where there is one cutoff equilibrium, the cutoff rule is infinite
2. For values of r where there are three cutoff equilibria, two involve finite cutoff rules and one involves an infinite cutoff rule
3. The case of with two cutoff equilibria (one finite and one infinite) is excluded because is a non-generic case

Why is there always an infinite cutoff equilibrium?

> I begin with the first claim. To see that, for all $r \in \mathcal{R}$, there is a Bayesian Equilibrium with no participation, consider a strategy profile with $a_i = 0$ for all s_i. The probability of regime change is zero. If a player were to deviate to participating, the probability of regime change would still be zero, since all individuals are measure zero. Thus, the payoff to deviating is $-k < 0$.

$\hat{s} = \infty$

Reformulating we can say that in this game for each r $(m, k, T, \sigma_\epsilon, \sigma_\theta)$ no matter how high is θ (so no matter the level of uncertainty) it has no effect on the probability of the regime falling.

If a player expects no one else will participate, he is sure that the revolution never occurs and so the best response is not to participate because $0 > -k$.

It is important to underline that there is not always $\hat{s} = -\infty$ because if a player expects that everyone will participate he will decide to participate only if $\theta > k$.

Open Questions

Since the participation rule is $s_i \geq \hat{s}$ the use of $\hat{s} = \infty$ guarantees in terms of cutpoint strategies that no citizen participates in the revolution because all are characterized by $s_i < \infty$.

1. Why should citizens in the presence of a finite equilibrium cutoff expect no one to participate?
2. $G^{\theta}(\hat{s} = \infty, t) = 0$ so formally $\hat{s} = \infty$ can be equilibrium cutoff if and only if $k = 0$, but at that point not participating would not be strictly dominant strategy

The larger equilibrium cutoff is not stable

The author states that in case of two equilibrium cutoffs only one is stable/plausible

"Consider the equilibrium where all players use the cutoff rule $\hat{s}^+$. If play is slightly perturbed such that a few too many players participate, then players with types slightly lower than $\hat{s}^+$ want to participate, making more players want to participate, until everyone with a type greater than $\hat{s}^-$ is participating. Similarly, if a few too few players participate, then players with types slightly higher than $\hat{s}^+$ do not want to participate, making more players not want to participate, until no one is participating"

Is it correct to discard $\hat{s}^+$?

1. The logic of this statement is valid if citizens participate in the revolution at different times
2. Their payoff and their belief in the probability of success of the revolution do not depend on the participation of the other players, so as participation increases, their intentions do not change
3. It is not clear why someone who is characterised by $s_i > \hat{s}^+$ should be persuaded not to participate by having an incremental benefit from doing so

Equilibrium smoothness and comparative statics

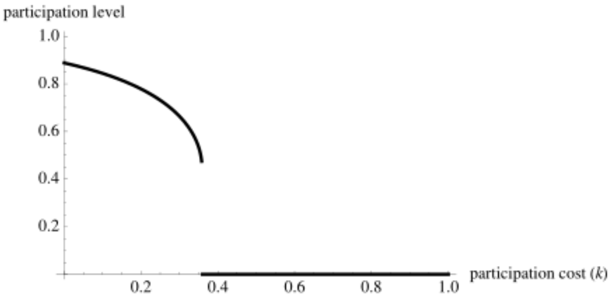
The equilibrium cutoff is defined by

$$G(\hat{s};r) \equiv \left(1 - \Phi\left(\frac{(1-\lambda)}{\sigma_\lambda}\hat{s} - \frac{(1-\lambda)m + \sigma_\epsilon \Phi^{-1}(1-T)}{\sigma_\lambda}\right)\right)$$
$$\times \left(\lambda\hat{s} + (1-\lambda)m + \sigma_\lambda \frac{\phi}{1-\Phi}\left(\frac{(1-\lambda)}{\sigma_\lambda}\hat{s} - \frac{(1-\lambda)m + \sigma_\epsilon \Phi^{-1}(1-T)}{\sigma_\lambda}\right)\right) = k.$$

$G^\theta(\hat{s}, r)$ is a function of $T, k, m, \sigma_\epsilon^2, \sigma_\theta^2$ therefore the equilibrium cutoff changes continuously with changes in these parameters

Spontaneous Revolution

This scenario is able to take account of spontaneous revolutions since there is a critical threshold in k after which there is only a zero participation equilibrium. Thus, around that threshold, a small change in costs of participation leads to a discontinuous change in participation



Threshold Uncertainty

Each player receives a **signal** $t_i = T + \epsilon_i$ where:

1. T is drawn from $N(m, \sigma_T^2)$
2. $\epsilon_i \sim N(0, \sigma_\epsilon^2)$
3. T, ϵ_i independent
4. θ is known

Define P the set of possible combinations of parameters $m, \sigma_T, \sigma_\epsilon, \theta, k$ where $\theta > k$.
We restrict the space to the case where $\theta > k$ because otherwise *not to participate* is always a dominant strategy regardless of signal.

$\Gamma^T(p)$ is a specific instance of the game

Threshold Uncertainty

Suppose player i believes all players j participate if and only if:

$$t_j \leq \hat{t} \iff \epsilon_j \leq \hat{t} - T$$

Then player i anticipates total participation:

$$P(\epsilon_j \leq \hat{t} - T) = \Phi\left(\frac{\hat{t} - T}{\sigma_\epsilon}\right)$$

Hence, player i believes regime change will be achieved if and only if $T \leq T^*(\hat{t}, p)$ with :

$$\Phi\left(\frac{\hat{t} - T^*(\hat{t}, p)}{\sigma_\epsilon}\right) = T^*(\hat{t}, p) \iff \hat{t} = \Phi^{-1}(T^*(\hat{t}, p))\sigma_\epsilon + T^*(\hat{t}, p)$$

Threshold Uncertainty

The posterior beliefs about T are:

$$T|t_i \sim N(\overline{m}_i = \gamma t_i + (1-\gamma)m, \sigma_\gamma^2 = \gamma\sigma_\epsilon^2) \quad \gamma = \frac{\sigma_T^2}{\sigma_T^2 + \sigma_\epsilon^2}$$

From the perspective of a player i the probability of regime change becomes:

$$P(T \leq T^*(\hat{t}, p)) = \Phi\left(\frac{T^*(\hat{t}, p) - \overline{m}_i}{\sigma_\gamma}\right)$$

Hence a player will participate if:

$$\left[\Phi\left(\frac{T^*(\hat{t}, p) - \gamma t_i - (1-\gamma)m}{\sigma_\gamma}\right)\right]\theta \geq k$$

Threshold Uncertainty

An equilibrium cutoff rule must satisfy

$$\left[\Phi\left(\frac{T^*(\hat{t},p)-\gamma\hat{t}-(1-\gamma)m}{\sigma_\gamma}\right)\right]\theta = k$$

$$G^T(\hat{t},p) = \left[\Phi\left(\frac{T^*(\hat{t},p)-\gamma\hat{t}-(1-\gamma)m}{\sigma_\gamma}\right)\right]\theta$$

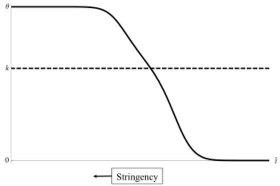
$$G^T(\hat{t},p) = \left[\Phi\left(\frac{(1-\gamma)(T^*(\hat{t},p)-m)-\gamma\Phi^{-1}(T^*(\hat{t},p))\sigma_\epsilon}{\sigma_\gamma}\right)\right]\theta$$

Obviously for any finite value of $\hat{t}$ $G^T(\hat{t},p)\in(0,\theta)$

Unique Equilibrium

The game $\Gamma^T(p)$ has always an **unique finite equilibrium** in the set P^* where is satisfied the condition

$$\sigma_\epsilon < \sigma_T^2 \sqrt{2\pi}$$



Because when this condition is met, the function is strictly decreasing in $\hat{t}$

However, it is **not a necessary condition for a single finite equilibrium** to be observed.

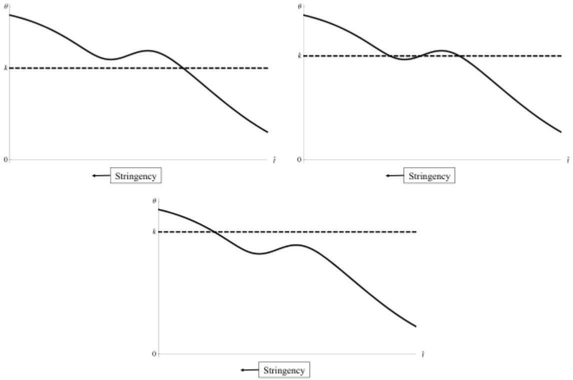
Thick Tails

The condition

$$\sigma_{\epsilon} < \sigma_T^2 \sqrt{2\pi}$$

can be interpreted as the **thick tails assumption**

Even in the case where the condition is not satisfied and therefore the function is not monotonic in $\hat{t}$, it is always possible to find an open set of k values for which the finite equilibrium is unique and an open set of k values where there are three finite equilibria, except for the non-generic case when we observe two equilibria



No infinite cutoff equilibrium

In this case **there is not an equilibrium with infinite cutoff**

If a player expects no one else will participate if $T < 0$ he is sure that the revolutions occurs and so since $\theta > k$ he will participate, while if he expects all the citizens participate if $T > 1$ he is sure that the revolution does not occur and therefore he will not participate

This is the key that provides uniqueness of equilibrium in the Γ^T game.

This assumption seems unrealistic.

If a single, measure zero person participates when $T < 0$, she derives the benefit θ when the regime falls, even though she was irrelevant to causing that event.

Hence, there is an important sense in which the assumptions underlying the non-existence of a zero-participation equilibrium in Γ^T are substantively less well motivated than the assumptions underlying Γ^θ .

This suggests another reason why equilibrium multiplicity, and thus arguments like Schelling's (1960) and Weingast's (1997), should remain an important feature of our understanding of mass uprisings and revolution

Discussion

The analyses of these two games suggests two ways in which such models can be consistent with arguments, like Schelling's (1960) and Weingast's (1997), that rely on equilibrium multiplicity

The game Γ^{θ} does not induce two-sided limit dominance, then the game naturally has multiple equilibria because it always has an equilibrium with no participation, in addition to admitting the possibility of a stable positive participation equilibrium

The game Γ^{T} can still have multiple equilibria, as long as two conditions hold

1. the participation costs k are neither too high nor too low
2. thin tails (σ_T^2 not too big)

However the condition that generates uniqueness of equilibrium seems unrealistic

Discussion

The game Γ^{θ} saves the smoothness of equilibrium behaviour from the global game and involves multiple equilibria according to the literature.

The game Γ^{T} is a valid alternative however the initial condition leading to uniqueness of equilibrium (in contrast to the literature) seems unrealistic and is not assumed in the previous game

There is no tension between arguments about mass uprisings that depend on equilibrium multiplicity and a modeling approach that introduces uncertainty in order to generate smoothness. As such, the analysis provided here highlights the importance of theorists introducing uncertainty into models of regime change in a substantively plausible way.

Increased Stringency

In order to compare the responses of G^T and G^θ to increases in stringency the author compares

$$-\frac{dG^T(\hat{t};p)}{d\hat{t}} = \phi\left(\frac{T^*(\hat{t};p)-\gamma\hat{t}-(1-\gamma)m}{\sigma_\gamma}\right)\frac{\theta}{\sigma_\gamma}\boxed{\left(\gamma-\frac{dT^*(\hat{t};p)}{d\hat{t}}\right)},$$

$$\frac{dG^\theta(\hat{s};r)}{d\hat{s}} = \phi\left(\frac{\theta^*(\hat{s};r)-\lambda\hat{s}-(1-\lambda)m}{\sigma_\lambda}\right)\frac{\left(\lambda\hat{s}+(1-\lambda)m+\sigma_\lambda\frac{\phi\left(\frac{\theta^*(\hat{s};r)-\lambda\hat{s}-(1-\lambda)m}{\sigma_\lambda}\right)}{1-\Phi\left(\frac{\theta^*(\hat{s};r)-\lambda\hat{s}-(1-\lambda)m}{\sigma_\lambda}\right)}\right)}{\sigma_\lambda}\boxed{\left(\lambda-\frac{d\theta^*(\hat{s};r)}{d\hat{s}}\right)}$$
$$+\left(1-\Phi\left(\frac{\theta^*(\hat{s};r)-\lambda\hat{s}-(1-\lambda)m}{\sigma_\lambda}\right)\right)\left(\lambda+\sigma_\lambda\frac{d\frac{\phi}{1-\Phi}}{d\hat{s}}\left(\frac{\theta^*(\hat{s};r)-\lambda\hat{s}-(1-\lambda)m}{\sigma_\lambda}\right)\right).$$

since increased stringency involves decreasing the cutoff rule in Γ^T but increasing the cutoff rule in Γ^θ .

Beliefs Effects

When the cutoff becomes more stringent both the games are characterized by the **beliefs effect**

When the cutoff rule is more stringent, a player whose signal equaled the cutoff rule received a better signal and so believes the state is more favorable to regime change

The beliefs effects in Γ^{θ} and Γ^{T} are represented by λ and γ, respectively. This reflects the fact that the more informative is the signal in either game, the larger is the beliefs effect. These magnitudes are unaffected by the stringency of the cutoff rule.

Critical-threshold Effect

When the cutoff becomes more stringent both the games are characterized by the **critical-threshold effect**

When the cutoff rule is more stringent, the true state of the world must be more favorable (i.e., must be lower) in order for regime change to be achieved

The critical-threshold effects in Γ^{θ} and Γ^{T} are represented by $-\frac{d\theta^*(\hat{s},r)}{d\hat{s}}$ and $-\frac{dT^*(\hat{t},p)}{d\hat{t}}$

Observation 1

The critical-threshold effect is larger in Γ^{θ} than in Γ^{T}

$$\frac{dT^*(\hat{t};p)}{d\hat{t}} = \frac{\frac{1}{\sigma_\xi}\phi\left(\frac{\hat{t}-T^*(\hat{t};p)}{\sigma_\xi}\right)}{\frac{1}{\sigma_\xi}\phi\left(\frac{\hat{t}-T^*(\hat{t};p)}{\sigma_\xi}\right)+1} < 1$$

$$\frac{d\theta^*(\hat{s};r)}{d\hat{s}} = \frac{\phi\left(\frac{\hat{s}-\theta^*(\hat{s};r)}{\sigma_\epsilon}\right)}{\phi\left(\frac{\hat{s}-\theta^*(\hat{s};r)}{\sigma_\epsilon}\right)} = 1.$$

Making the state of the world more favorable in the game Γ^T means a lower realization of T. Such a change has two effects

1. more people receive a good enough signal to cross the cutoff rule, increasing participation
2. fewer people need to participate in order to achieve regime change

The second effect does not have an analogue in the game Γ^θ

For any given incremental increase in the stringency of $\hat{t}$, a decrease in T^* that is of a smaller size than the increase in $\hat{t}$ will continue to assure regime change

Observation 2

The critical-threshold effect is larger than the beliefs effects in Γ^θ

$$\lambda = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\epsilon^2} < 1$$

Taking into account only these two effects, increasing the stringency of the cutoff rule, therefore, always makes the player whose signal is at the cutoff rule worse off

Observation 3

The critical-threshold effect in Γ^T becomes negligible as stringency increases

$$\lim_{\hat{t} \to -\infty} \frac{dT^*(\hat{t}, p)}{d\hat{t}} = 0$$