

Exercise

The diagnostic test

Exercise 1

In a certain population the probability of having a given disease (M) is 0.85. The diagnostic test generally used has a probability of 0.8235 to recognize this disease, and therefore to be positive, if a person is actually ill. The probability that a healthy person is positive for the test is $\frac{2}{3}$. From this population a sample of 500 people is extracted and they are submitted to the diagnostic test. How many of the test negative people will be really healthy?

Answer

$$P(M+) = 0.85 \Rightarrow P(M-) = 0.15$$

$$P(T+|M+) = 0.8235$$

$$P(T+|M-) = 2/3$$

$$n = 500$$

$$\# M-|T- = ?? = 500 * P(T- \cap M-)$$

$$\#(M+) = 500 * 0.85 = 425 \Rightarrow \#(M-) = 500 - 425 = 75$$

$$P(T+ \cap M-) = P(T+|M-) * P(M-) =$$

$$= 2/3 * 0.15 = 0.10 \Rightarrow P(T- \cap M-) = 0.15 - 0.10 = 0.05$$

$$e \# M-|T- = ?? = 500 * 0.05 = 25$$

	M+	M-	Total
T+			
T-		25	
Total	425	75	500

Exercise 2

A diagnostic test for disease M has 95% specificity and sensitivity. The test is applied to a population of 4000 subjects in which the prevalence of M disease is 6%. How many subjects are expected to be positive for the test?

Answer

$$Se = P(T+|M+) = 0.95$$

$$Sp = P(T-|M-) = 0.95$$

$$n = 4000$$

$$P(M+) = 0.06$$

T+=??

$$\#(M+) = 4000 * 0.06 = 240 \Rightarrow \#(M-) = 4000 - 240 = 3760$$

The number of positive tests among patients will be:

$$\#(T+|M+) = 0.95 * 240 = 228$$

The number of positive test results among the healthy will be:

$$\#(T+|M-) = 0.05 * 3760 = 188$$

$$\# T+ = 228 + 188 = 416$$

	M+	M-	Total
T+	228	188	416
T-			
Total	240	3760	4000

Exercise 3

In a screening test for the early diagnosis of the M disease, 110 of the 1000 subjects tested were positive, but 45 of these were not really sick. If the prevalence of the disease M in the population examined is 10%:

- a) How many false negatives are there?
- b) What is the sensitivity of the test?
- c) What is the predictive value of a positive test result?

Answer

$$P(M+) = 0.10 \Rightarrow \# M+ = 100$$

a) The false negative are

b) The sensitivity of test is:

$$65/100 = 0.65$$

c) $VPP = P(M+|T+) = 65/110 = 0.59$

	M+	M-	Total
T+	65	45	110
T-	35	855	890
Totale	100	900	1000

Exercise 4

A group of researchers wants to evaluate a certain screening test for Alzheimer's. The test included two samples, one consisting of 450 pieces randomly selected with Alzheimer's disease and another 500 pieces that did not show the symptoms of the disease.

Test result	Alzheimer diagnostic		
	Yes (D+)	No(D-)	Total
Positive	436	5	441
Negative	14	495	509
Total	450	500	950

- Estimate the sensitivity and specificity of the test
- If the probability of disease in the population is 11.3%, what is the PPV?

Solution

- Sensitivity: $436/450=0.9689$
- Specificity: $495/500=0.99$
- $PPV = \frac{Se * P(D+)}{[if * P(D+) + (1-Sp) * P(D-)]} =$
 $= \frac{0.9689 * 0.113}{0.9689 * 0.113 + 0.01 * (1 - 0.113)} =$
 $= \frac{0.10949}{0.10949 + 0.00887} = 0.925$

Exercise 5

Verna et al. They examined the use of heparin-PF4 ELISA screening to test heparin-induced thrombocytopenia (HIT) in critically ill patients. Using C-serotonin release (SRA) measurements as a means to validate HIT, the authors found that 31 patients were negative for SRA, of which 22 were negative for heparin-PF4 ELISA.

- a) Calculate the specificity of heparin-PF4 ELISA for HIT
- b) Using the sensitivity found in the literature of 95% and a probability of having HIT of 3.1%, what is the PPV?
- c) Using the information in point b), what is the NPV?

Solutions

	HIT		
	SRA+	SRA-	Total
Heparin-PF4 ELISA+			
Heparin-PF4 ELISA-		22	
Total		31	

- $Sp = 22/31 = 71\%$
- $Se = 0.95$ $P(\text{HIT}) = 3.1\%$
 $PPV = Se \cdot p / [Se \cdot p + (1 - Sp) \cdot (1 - p)]$
 $0.95 \cdot 0.031 / (0.95 \cdot 0.031 + (1 - 0.71) \cdot (1 - 0.031)) =$
 $0.02995 / (0.02995 + 0.29 \cdot 0.969) = 0.02995 / (0.02995 + 0.28101) =$
 $0.095 = 9.5\%$
- $NPV = Sp \cdot (1 - p) / [Sp \cdot (1 - p) + (1 - Se) \cdot p]$
 $0.71 \cdot 0.969 / [0.71 \cdot 0.969 + 0.05 \cdot 0.031] =$
 $0.68799 / (0.68799 + 0.00155) = 0.68799 / 0.68954 =$
 $0.998 = 99.8\%$

Exercise 6

The first test used in screening for HIV infection was the ELISA test. In a population of 5,000 individuals tested, 20 were infected by HIV but were test negative, 980 were infected and were positive for testing, 8 were not infected but were positive for the test and 3992 were not infected and were negative for the test .

- a) What was the probability of being positive for the healthy test?
- b) What was the probability of being negative for the test among the infected?
- c) If a random individual chosen by this population is positive for the test, what is the probability of having HIV?
- d) What is the probability of being mis-classified by the test?

Solutions

Test ELISA	HIV		Total
	+	-	
T+	980	8	988
T-	20	3992	4012
Total	1000	4000	5000

a) $P(T+|HIV-)=8/4000=0.002$

b) $P(T-|HIV+)=20/1000=0.02$

c) $P(HIV+|T+)=980/988=0.9919=99.2\%$

d) $(20+8)/5000=0.0056=0.56\%$

Exercise 7

Of the 1820 subjects in one study, 30 suffered from tuberculosis, 1790 no. All subjects were subjected to chest radiography; 73 presented a positive X-ray, while the other 1747 were negative.

a) What is the probability that a person randomly selected by the general population is affected by tuberculosis since his radiography is positive and that the prevalence of tuberculosis in the population is 0.000093 (0.0093%)?

Radiography	Tuberculosis		Total
	+	-	
+	22	51	73
-	8	1739	1747
Total	30	1790	1820

Solutions

$$Se=22/30=0.7333$$

$$Sp=1739/1790=0.9715$$

$$PPV=Se*P/[Se*P+(1-Sp)*(1-P)]=$$

$$=0.7333*0.000093/ [0.7333*0.000093+0.0285*0.999907]=$$

$$= 0.0000681969 / [0.0000681969+0.0284973495]=$$

$$= 0.0000681969 / 0.0285655464= 0.00239=0.239\%$$