Principles of Corporate Finance

Written exam (proctored online) - July 13, 2020

THE EXAM LASTS 1 HOUR

THOSE WHO HAVE PRESENTED IN CLASS MUST ANSWER THE 2 NUMERICAL QUESTIONS.
ALL OTHERS HAVE 15 MINUTES MORE AND MUST ANSWER ALSO THE OPEN QUESTION.

Question 1 (numerical)

The entrepreneur E owns liquidity A and seeks external funding for an investment that requires $I = 50$ at $t = 0$ and that returns $X = \{0, 100\}$ at $t = 2$. E can choose between two projects: a good project H and a bad project L. The success probability is $\Pr\{X = 100\} = p$; project H has a greater success probability $p_H = 0.8$, while project L has $p_L = 0.3$. However project L guarantees to E a private benefit $B = 40$.

1. E raises $(I - A)$ by issuing a bond that repays a face value R_u to investors. Write the incentive constraint for E to choose project H and compute his maximum pledgeable income (constraint on R_u). Write the investors' rationality constraint and find the minimum value R_u, assuming that E chooses project H. Find the minimum threshold for A for which E manages to raise external financing.

2. The bank monitors at cost $c = 2$, reducing as a consequence the private benefit from $B = 40$ to $b = 20$. Assume an E who is credit rationed. E asks funding exclusively to a bank and promises to repay R_m at $t = 2$. Which is the minimum threshold for A to obtain a loan from the bank?

3. Assume now own funds A are uniformly distributed between 0 and 100, that is A has density $h(A) = \frac{1}{100}$ on the interval $[0, 100]$. Compute the percentage of firms that are credit rationed, those that are financed by financial markets, those financed by the banks and those that self-finance the investment.

Question 2 (numerical)

Consider an entrepreneur E who owns an asset in place at $t = 0$ that will return a cash flow at $t = 2$: the cash flow will be $X^+ = 100$ if E is of type H, while $X^- = 50$ if E is of type L. At $t = 1$ there is a new investment opportunity: by investing $I = 20$ at $t = 1$ this project will return $Y = 30$ at $t = 2$ by sure. E has no funds to finance this new opportunity, hence he has to issue stocks on competitive financial markets.

1. Assume new investors observe the type of E: which fraction $(1 - \alpha) \in (0, 1)$ of the cash flow will investors request to finance a firm of type H? Which fraction to finance firm of type L?

2. Assume now that new investors do not observe the type of E. The probability that E is of type H is $q = 0.1$. If new investors expect that both type of E will invest, which fraction $(1 - \hat{\alpha})$ of cash flow must be promised to investors in order to convince them to finance the firm? Do you think investors' belief are correct at the equilibrium?

3. Assume now that investors expect that only type L will invest. Which fraction $1 - \hat{\alpha}$ must be promised to new investors in this case? Are the expectations correct at the equilibrium?

Question 3*

Discuss the reasons why conglomerates might be traded at a discount in financial markets.

Solutions for the numerical questions

Question 1

1. E will choose project H whenever

$$.8 \times (100 - R_u) \geq .3 \times (100 - R_u) + 40 \Leftrightarrow R_u \leq 20$$

Bondholders will finance E if and only if

$$.8 \times R_u + .2 \times 10 \geq 50 - A \Leftrightarrow R_u \geq \frac{48 - A}{0.8}$$

Combining the two inequalities, we have that

$$\frac{48 - A}{0.8} \leq 20$$

Hence the minimum level of A fulfilling the above condition is

$$\bar{A} = 32$$

2. When the bank finances him, E wll chose project H if and only if

$$.8 \times (100 - R_m) \geq .3 \times (100 - R_m) + 20 \Leftrightarrow R_m \leq 60$$

The bank will finance E if and only if

$$.8 \times R_m + .2 \times 10 - 15 \geq 50 - A \Leftrightarrow R_m \geq \frac{63 - A}{0.8}$$

Combining the two inequalities, we have that

$$\frac{63 - A}{0.8} \leq 60$$

Hence the minimum level of A fulfilling the above condition is

$$\underline{\mathrm{A}} = 15$$

3. With a uniform distribution between 0 and 100, we have that:

- 50% self-finance their investment
- 18% finance by issuing bonds in financial markets
- 17% are financed by banks
- 15% are credit rationed

Question 2

1. To finance type H, investors require:

$$(1 - \alpha_H)(100 + 30) \geq 20 \quad (1)$$

that is $(1 - \alpha_H) = 20/30 = 0.1538$. To finance type L instead

$$(1 - \alpha_L)(50 + 30) \geq 20 \quad (2)$$

that is $(1 - \alpha_L) = 1/4 = 0.25$.

2. When investors do not observe E's type, they ask a single $1 - \hat{\alpha}$ equal for both types of E, to fulfill the rationality condition:

$$(1 - \hat{\alpha})[0.1(100 + 30) + 0.9(50 + 30)] \geq 20 \quad (3)$$

thus, when the constraint is binding $1 - \hat{\alpha} = 20/85 = 0.2352$. Let us check that type H will invest (type L is more likely to invest once type H invests). Type H invests if and only if

$$\hat{\alpha}(100 + 30) \geq 90 \quad (4)$$

Substituting from $1 - \hat{\alpha} = 0.2352$, the inequality does not hold. Hence type H will not invest at the equilibrium. It is easy to check that type L will invest. Hence expectations are wrong at the equilibrium. The equilibrium is a separating equilibrium (the two types behave differently).

3. Investors expect the fraction to fulfill the rationality condition

$$(1 - \hat{\alpha})(50 + 30) \geq 20 \quad (5)$$

from which $(1 - \hat{\alpha}) = 0.25$. We must check if type H invests when $\hat{\alpha} = 0.75$

$$(1 - 0.25)(100 + 30) \geq 100 \quad (6)$$

since type H does not invests, there is only a separating equilibrium in which only type L invests.