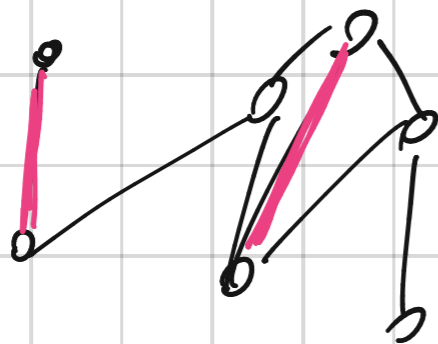
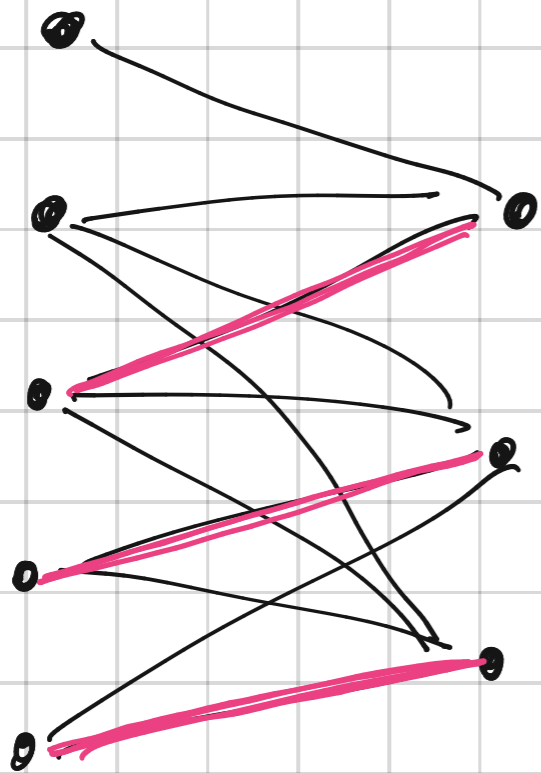


Matching  $M \subseteq E$  such that each vertex  $v \in V$  is incident on at most one edge of  $M$ .



Problem: Find a largest (maximum) matching in  $G$

# Bipartite graph



Let  $X \subseteq L$  where  $(L, R)$  is the bipartition of  $G$

A matching  $M$  is  $X$ -saturating if all vertices of  $X$  are incident on some edge of  $M$

## Hall's Theorem

Let  $G$  be a bipartite graph with bipartition  $(L, R)$ , and let  $X \subseteq L$ .

Then there exists an  $X$ -saturating matching  $M$  iff  $|N(W)| \geq |W|$  for each  $W \subseteq X$ .

## Proof (constructive)

We maintain:

- $M$  matching
- $X^k$  set of marked vertices of  $L$  at iteration  $k$
- $Y^k$  " " " " " " " " " " " "