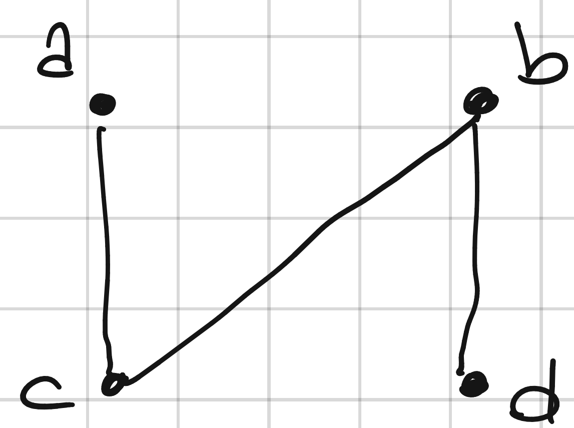
Graph $G = (V, E)$

- V : vertices
- E : edges

$E \subseteq V \times V$

$(v, w) \in E \Rightarrow (w, v) \in E$

Undirected graph



$(v, v) \in E$ loop

Graph coloring

Instance

Graph $G=(V,E)$

Feasible solutions

$c: V \mapsto L$ such that $\forall (v,w) \in E$, $c(v) \neq c(w)$

Decision version

$k \in \mathbb{N}$ part of the problem

$|L| \leq k$ Does a coloring c exist?

Each planar graph is 4-colorable

3-colorability is NP-hard on planar graphs

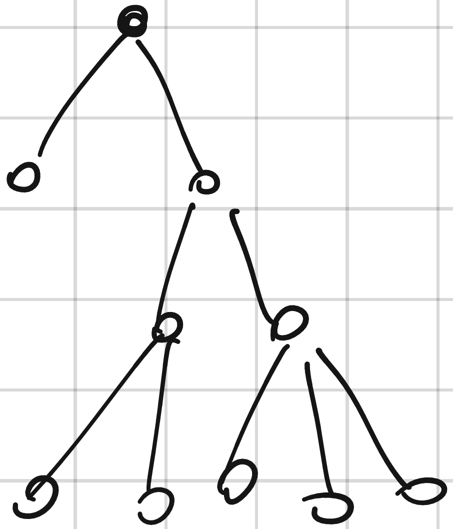
Vertex Cover

$C \subseteq V$ is a cover for a graph $G=(V,E)$

if $\forall (v,w) \in E$, $v \in C \vee w \in C$

NP-hard

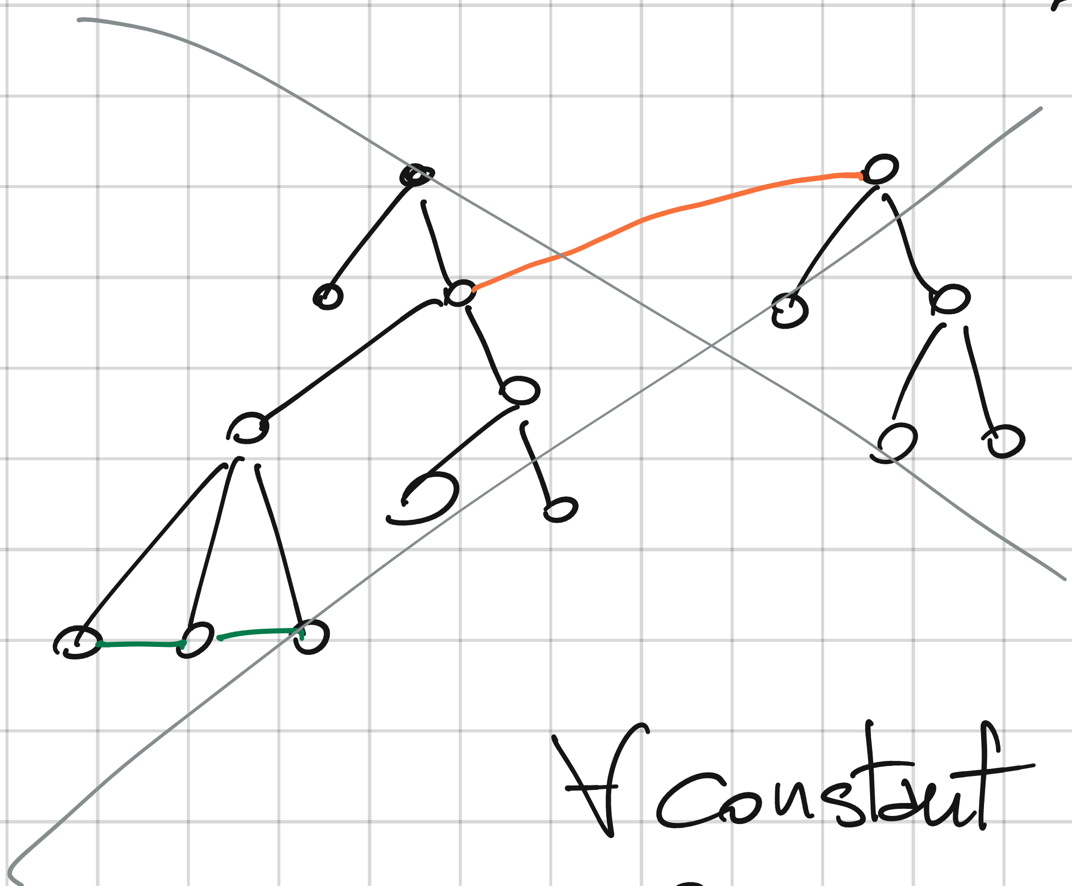
Problems on trees are usually simpler



Dynamic Programming

Min Weighted Vertex Cover

Trees $\subseteq$ Partial k-trees $\subseteq$ Graphs



Treewidth

Graphs with treewidth $\leq k$

$\forall$ constant $k \in \mathbb{N}$, the set of partial k-trees has a dynamic programming polytime for Min Weighted Vertex Cover

Treewidth

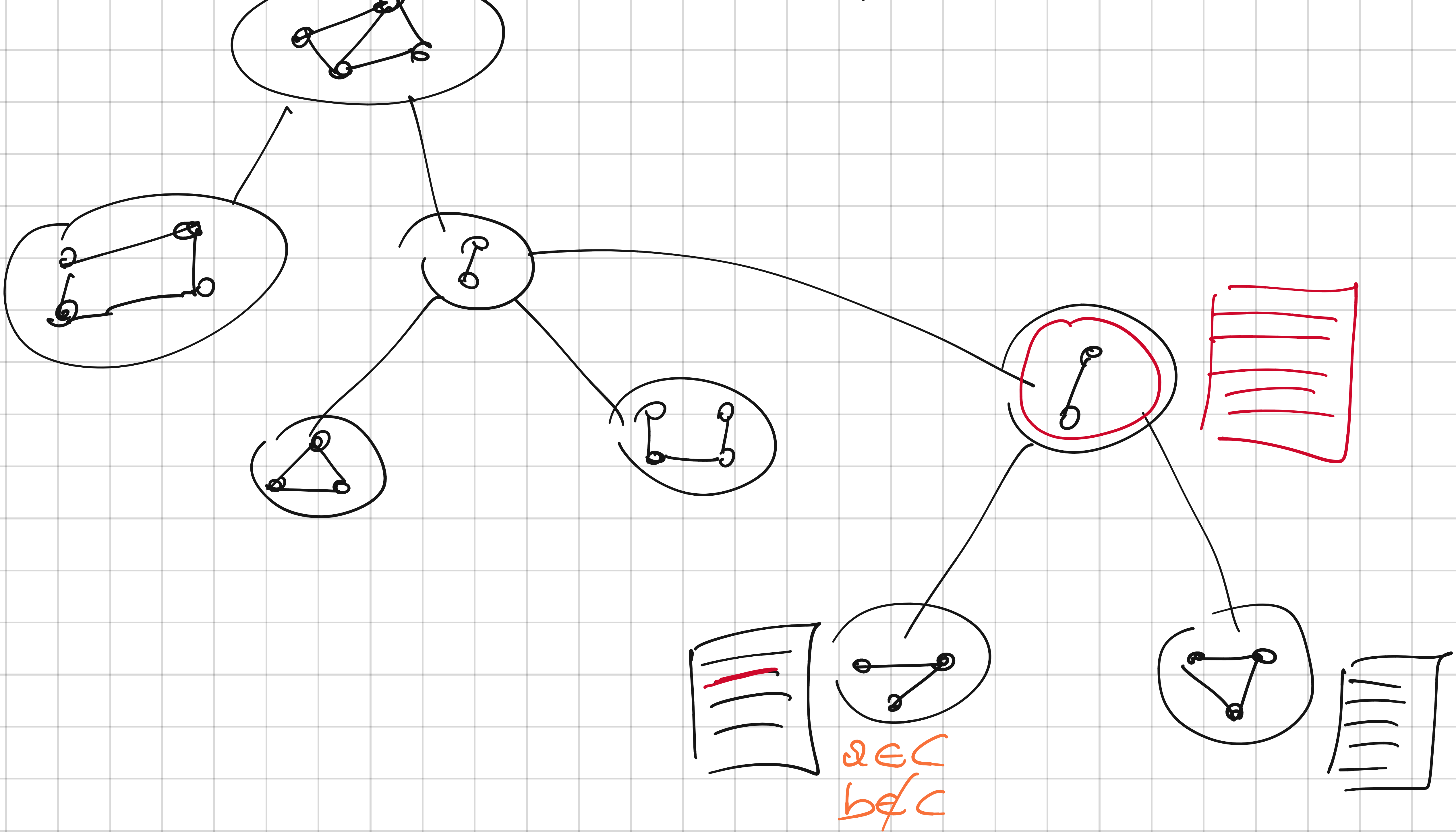
Collection $X_1, \dots, X_p$ of bags (subset of V)

there exists a tree T with vertices X_i

$\bigcup X_i = V$

- If X_i and X_j have a common vertex v
  and X_k is on the path from X_i to X_j on T
  Then $v \in X_k$
- If $(v,w) \in E \Rightarrow \exists X_i$ s.t. $v, w \in X_i$
  $tw(G) = \max\{|X_i| - 1\}$

Partial k-tree



Clique-width

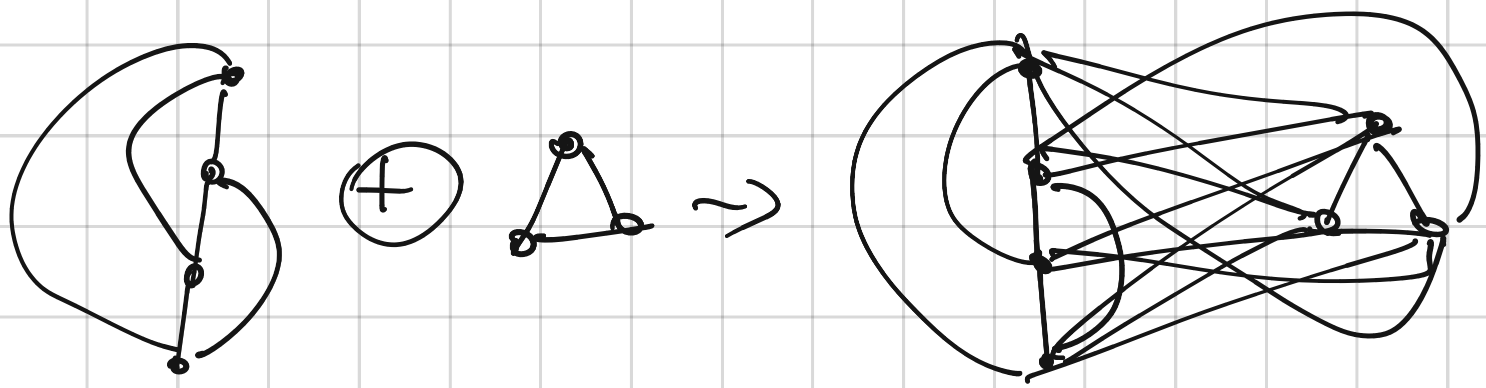
Almost a cograph

Cograph :

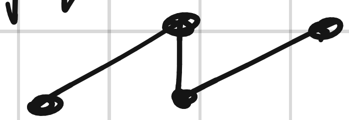
A graph obtained with 2 operation

1) Disjoint union

2) Clique-sum

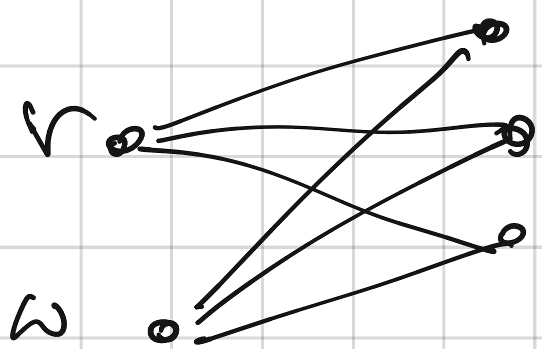


A graph G is a cograph iff G does not have an induced P_4



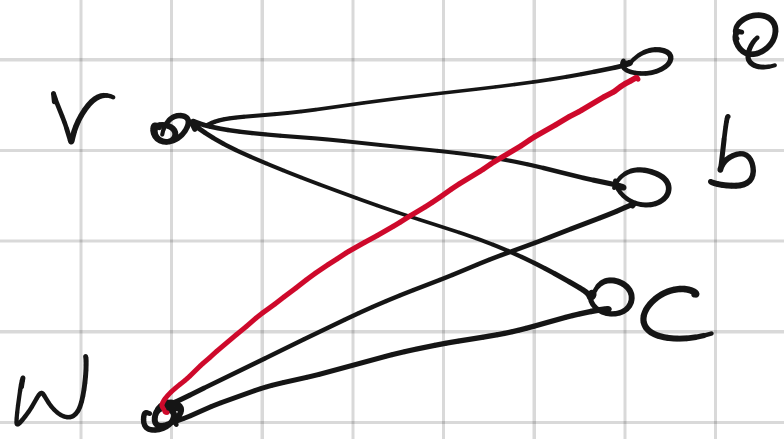
Twin-width

G is a cograph if there exists a twin for some vertex v



$$N(v) = N(w)$$

Twin-width 2



w is a near twin of v

twin-width of a graph G is the smallest of red edges that have to be added at each step

Courcelle's Theorem

Let P be a property that can be expressed in monadic second order graph logic (MSO)

If G has constant treewidth $\Rightarrow$

P can be tested in linear time.

MSO:

- relation $adj(v, w)$
- Property P is stated in terms of sets of vertices

Vertex Cover

$$\exists C \subseteq V : \forall v \ \forall w \quad adj(v,w) \Rightarrow (v \in C \lor w \in C)$$

3-colorability

$$\exists C_1 \subseteq V, \exists C_2 \subseteq, \exists C_3 \subseteq V$$

$$\forall a \ (a \in C_1 \lor a \in C_2 \lor a \in C_3)$$

$$\forall a \ \forall b \quad a \in C_1 \land b \in C_1 \Rightarrow \neg adj(a,b)$$

$$\forall a \ \forall b \quad a \in C_2 \land b \in C_2 \Rightarrow \neg adj(a,b)$$

$$\forall a \ \forall b \quad a \in C_3 \land b \in C_3 \Rightarrow \neg adj(a,b)$$

Algorithm for maximum matching

- Assignment 1: describe the time complexity of the algorithm