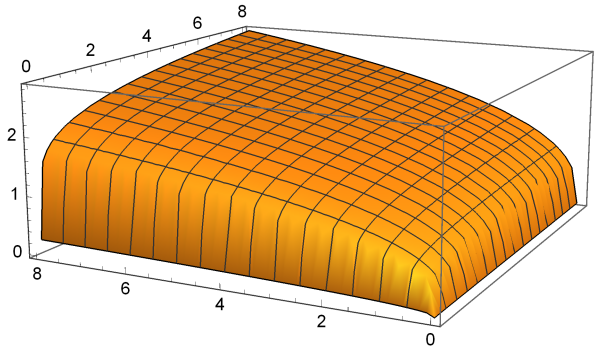
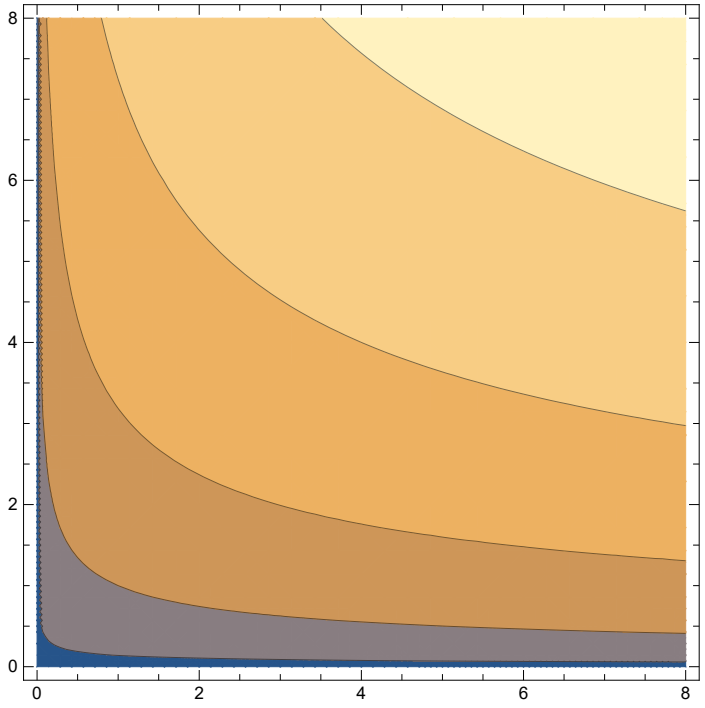
U = $\left(X^{0.3} * Y^{0.7}\right)^{\frac{1}{2}}$

$\sqrt{X^{0.3}\, Y^{0.7}}$

Plot3D[U, {X, 0, 8}, {Y, 0, 8}]



ContourPlot[U, {X, 0, 8}, {Y, 0, 8}]



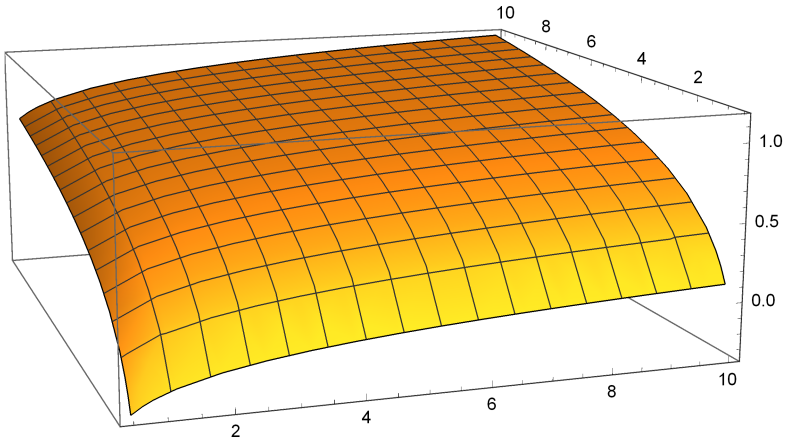
u = Log[U]

$\text{Log}\left[\sqrt{X^{0.3}\, Y^{0.7}}\right]$

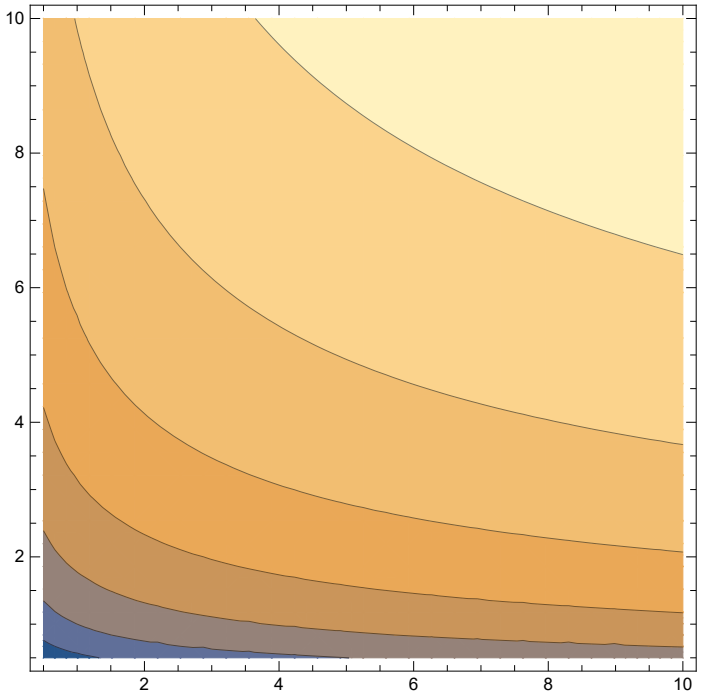
Simplify[u]

$\frac{1}{2}\,\text{Log}\left[X^{0.3}\, Y^{0.7}\right]$

```
Plot3D[u, {X, 0.5, 10}, {Y, 0.5, 10}]
```




```
ContourPlot[u, {X, 0.5, 10}, {Y, 0.5, 10}]
```




```
H = {{∂X,X u, ∂X,Y u}, {∂Y,X u, ∂Y,Y u}}
```

$$\left\{\left\{-\frac{0.15}{X^{2.}}, 0\right\}, \left\{0, -\frac{0.35}{Y^{2.}}\right\}\right\}$$

```
Eigenvalues[H]
```

$$\left\{-\frac{0.35}{Y^{2.}}, -\frac{0.15}{X^{2.}}\right\}$$

```
MatrixForm[H]
```

$$\begin{pmatrix} -\frac{0.15}{X^2} & 0 \\ 0 & -\frac{0.35}{Y^2} \end{pmatrix}$$

```
Det[H]
```

$$\frac{0.0525}{X^2\,Y^2}$$

NegativeSemidefiniteMatrixQ[H]

False or True if X, Y << ∞

Λ = u – λ (x * X + y * Y – R) con x e y prezzi

$$-(-R + x\,X + y\,Y)\,\lambda + \text{Log}\left[\sqrt{X^{0.3}\,Y^{0.7}}\right]$$

$\partial_X \Lambda$
$\partial_Y \Lambda$
$\partial_\lambda \Lambda$

$$\frac{0.15}{X^{1.}} - x\,\lambda$$

$$\frac{0.35}{Y^{1.}} - y\,\lambda$$

$$R - x\,X - y\,Y$$

$$\text{Solve}\left[\frac{0.15}{X^1} - x\,\lambda == 0 \;\&\&\; \frac{0.35}{Y^1} - y\,\lambda == 0 \;\&\&\; R - x\,X - y\,Y == 0,\ \{X, Y, \lambda\}\right]$$

$$\left\{\left\{X \to \frac{0.3`\,R}{x},\ Y \to \frac{0.7`\,R}{y},\ \lambda \to \frac{0.5`}{R}\right\}\right\}$$

Adding – up

$$\text{Evaluate}\left[x * \frac{0.3\,R}{x} + y * \frac{0.7\,R}{y} - R == 0\right]$$

True

Elasticità ai 2 prezzi e ad R di X

$$\text{Evaluate}\left[\left(\partial_x \frac{0.3\,R}{x}\right) * \frac{x}{\frac{0.3\,R}{x}} + \left(\partial_y \frac{0.3\,R}{x}\right) * \frac{y}{\frac{0.3\,R}{x}} + \left(\partial_R \frac{0.3\,R}{x}\right) * \frac{R}{\frac{0.3\,R}{x}} == 0\right]$$

True

Engel

$$\text{Evaluate}\left[\left(\partial_R \frac{0.3\,R}{x}\right) * x + \left(\partial_R \frac{0.7\,R}{y}\right) * y == 1\right]$$

True

Cournot

$$\text{Evaluate}\left[\left(\frac{x * \frac{0.3\,R}{x}}{R} * \left(\partial_x \frac{0.3\,R}{x}\right) * \frac{x}{\frac{0.3\,R}{x}}\right) + \left(\frac{y * \frac{0.7\,R}{y}}{R} * \left(\partial_x \frac{0.7\,R}{y}\right) * \frac{x}{\frac{0.7\,R}{y}}\right) == -\frac{x * \frac{0.3\,R}{x}}{R}\right]$$

True

Utilità indiretta e Roy

$$V = \frac{1}{2}\,\text{Log}\left[\left(\frac{0.3 * R}{x}\right)^{0.3} * \left(\frac{0.7 * R}{y}\right)^{0.7}\right]$$

$$\frac{1}{2}\,\text{Log}\left[0.542881\left(\frac{R}{x}\right)^{0.3}\left(\frac{R}{y}\right)^{0.7}\right]$$

$$-\frac{\partial_x V}{\partial_R V}$$

$$\left(0.162864\,R\left(\frac{R}{y}\right)^{0.7}\right) \Big/ \left(\left(\frac{R}{x}\right)^{0.7} x^2 \left(\frac{0.162864\left(\frac{R}{y}\right)^{0.7}}{\left(\frac{R}{x}\right)^{0.7} x} + \frac{0.380017\left(\frac{R}{x}\right)^{0.3}}{\left(\frac{R}{y}\right)^{0.3} y}\right)\right)$$

$$\text{Simplify}\left[\frac{0.162864\,R\left(\frac{R}{y}\right)^{0.7}}{\left(\frac{R}{x}\right)^{0.7} x^2 \left(\frac{0.162864\left(\frac{R}{y}\right)^{0.7}}{\left(\frac{R}{x}\right)^{0.7} x} + \frac{0.380017\left(\frac{R}{x}\right)^{0.3}}{\left(\frac{R}{y}\right)^{0.3} y}\right)}\right]$$

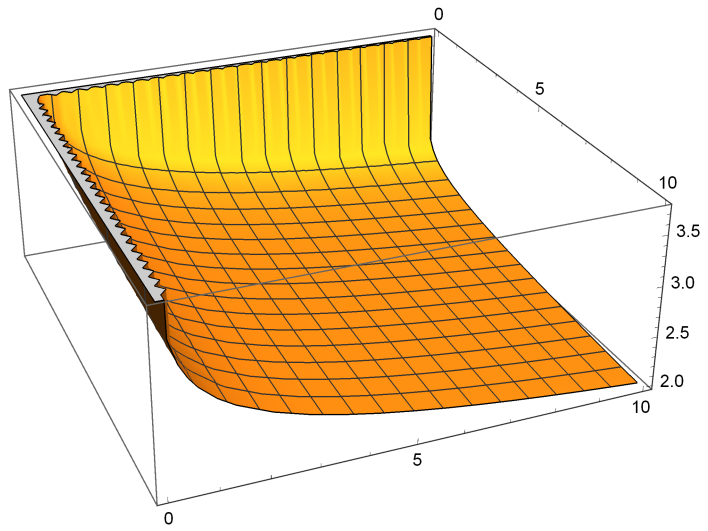
$$\frac{0.3\,R}{x}$$

Pongo R = 1000 per plottare la funzione

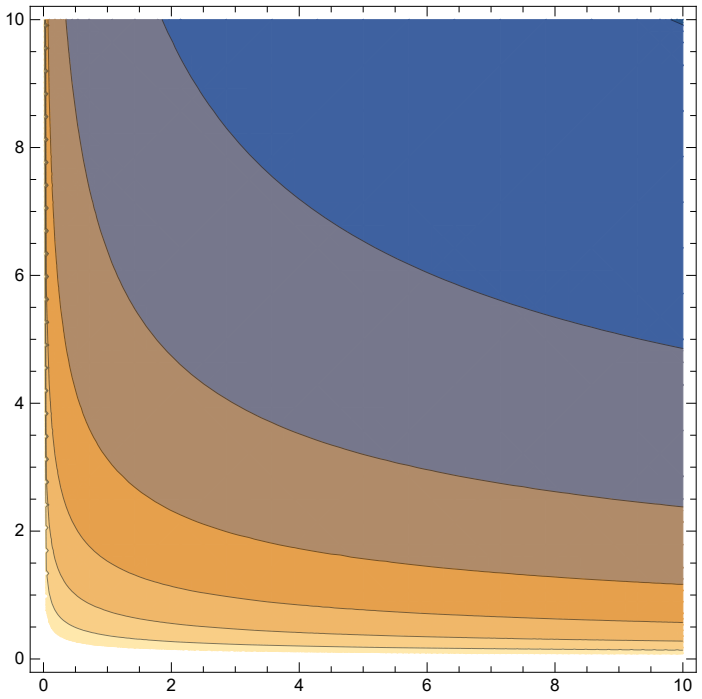
$$v = \frac{1}{2}\,\text{Log}\left[\left(\frac{0.3 * 1000}{x}\right)^{0.3} * \left(\frac{0.7 * 1000}{y}\right)^{0.7}\right]$$

$$\frac{1}{2}\,\text{Log}\left[542.881\left(\frac{1}{x}\right)^{0.3}\left(\frac{1}{y}\right)^{0.7}\right]$$

```
Plot3D[v, {x, 0, 10}, {y, 0, 10}]
```



ContourPlot[v, {x, 0, 10}, {y, 0, 10}]



$\mathcal{L}$ = x * X + y * Y − λ (u − W) con W = log $\bar{U}$ utilità fissata

$$x\,X + y\,Y - \lambda\left(-W + \text{Log}\left[\sqrt{X^{0.3}\,Y^{0.7}}\right]\right)$$

$\partial_X \mathcal{L}$

$$x - \frac{0.15\,\lambda}{X^{1.}}$$

$\partial_Y \mathcal{L}$

$$y - \frac{0.35\,\lambda}{Y^{1.}}$$

$\partial_\lambda \mathcal{L}$

$$W - \text{Log}\left[\sqrt{X^{0.3}\,Y^{0.7}}\right]$$

Domande compensate :

$$\mathbf{X\ (comp)\ =\ \left(\left(\frac{W}{1.34}\right)^2 * \left(\frac{y}{x}\right)^{0.7}\right)}$$

$$\mathbf{Y\ (comp)\ =\ \left(\left(\frac{W}{0.88}\right)^2 * \left(\frac{x}{y}\right)^{0.3}\right)}$$

da cui la funzione di spesa :

$$\mathbf{S = x * \left(\left(\frac{W}{1.34}\right)^2 * \left(\frac{y}{x}\right)^{0.7}\right) + y * \left(\left(\frac{W}{0.88}\right)^2 * \left(\frac{x}{y}\right)^{0.3}\right)}$$

$$1.29132\,W^2\left(\frac{x}{y}\right)^{0.3} y + 0.556917\,W^2\,x\left(\frac{y}{x}\right)^{0.7}$$

Simplify[S]

$$W^2 \left(1.29132 \left(\frac{x}{y}\right)^{0.3} y + 0.556917\, x \left(\frac{y}{x}\right)^{0.7}\right)$$

s = ∂_xS

$$\frac{0.387397\, W^2}{\left(\frac{x}{y}\right)^{0.7}} - \frac{0.389842\, W^2\, y}{x \left(\frac{y}{x}\right)^{0.3}} + 0.556917\, W^2 \left(\frac{y}{x}\right)^{0.7}$$

FullSimplify[s]

$$W^2 \left(\frac{0.387397}{\left(\frac{x}{y}\right)^{0.7}} + 0.167075 \left(\frac{y}{x}\right)^{0.7}\right)$$

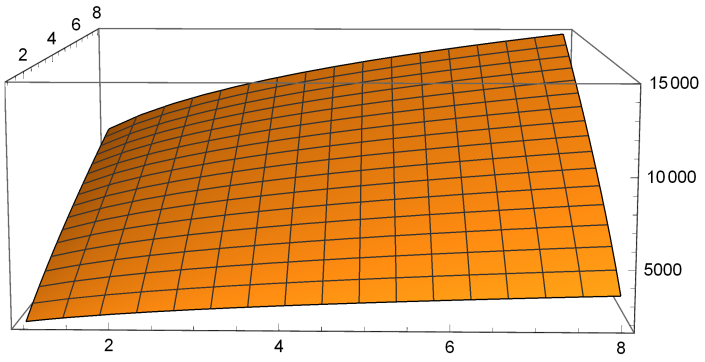
che corrisponde alla domanda compensata di X (vedi sotto per la corrispondenza della somma 0.5569...= 0.3873.. + 0.1670..).

Con W^2 = 1000

S = 1000 * $\left(1.29 * \left(\frac{x}{y}\right)^{0.3} * y + 0.55 * x * \left(\frac{y}{x}\right)^{0.7}\right)$

$$1000 \left(1.29 \left(\frac{x}{y}\right)^{0.3} y + 0.55\, x \left(\frac{y}{x}\right)^{0.7}\right)$$

Plot3D[S, {x, 1, 8}, {y, 1, 8}]



N[$\frac{1}{1.34^2}$]

0.556917