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A DISTANCE FUNCTION APPROACH TO PRICE EFFICIENCY

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1. Introduction

Recently there has been increased interest in price efficiency, where 'Price efficiency is attained if all maraginal rates of technical substitution are equated to the corresponding ratios of market prices for inputs' [Atkinson and Halvorsen (1986, p. 290)]. Deviation between marginal rates of technical substitution and relative prices may arise due to regulation, as in Atkinson and Halvorsen (1980, 1984, 1986), as well as due to other types of noncompetitive environments [see, for example, Toda (1976), Eakin and Kniesner (1988) and Lau and Yotopoulos (1971)]. We note from the above references that various functional forms as well as various behavioral assumptions are employed in the analysis of price efficiency.

To place our work in perspective, we first give a brief summary of the shadow price model used in these earlier studies. In particular, we follow the presentation from Atkinson and Halvorsen (1984, 1986). Denote inputs by $x=(x_1,\ldots,x_N)\in R^N_+$, outputs by $u=(u_1,\ldots,u_M)\in R^M_+$ and let $L(u)=\{x\in R^N_+$: x can produce $u\in R^M_+\}$[1] be the input set, i.e. the set of all input vectors that can produce the output vector u. In the shadow price model, firms are assumed to minimize the total (shadow) cost of producing a given output vector $u\in R^M_+$ for some shadow price vector $p^s=(p^s_1,\ldots,p^s_N)\in R^N_+$, i.e.

$$C^s(u,p^s)=\min_x\{p^s x: x\in L(u)\}=p^s x(u,p^s).$$

Now if market prices are $(p^o_1,\ldots,p^o_N)$, then the quotients of p^s_n and p^o_n, $n=1,\ldots,N$, may be used to define factors of proportionality, $k_n=p^s_n/p^o_n$, $n=1,\ldots,N$, which can in turn be used to judge price efficiency. In terms of the vector $k=(k_1,\ldots,k_N)$, price efficiency is attained if and only if $k_n=k_1$ for all $n=1,\ldots,N$.

Atkinson and Halvorsen (1986, p. 288) note that in theory the k_n,

[1]We assume that $L(u)$ satisfies: (a) $0\notin L(u)$, $u\geqq 0$, $u\neq 0$; (b) $x\geqq y\in L(u)\Rightarrow x\in L(u)$; (c) $L(u)$ is convex; (d) $L(u)$ is closed; and (e) $L(\theta u)\subseteqq L(u)$, $\theta\geqq 1$.

$n=2,\ldots,N$, factors of proportionality are input and firm specific. In practice, however, 'it is obviously not possible to identify separate values of the k_i's for each observation' (p. 289). The purpose of this short note is to show how separate values of the k_i's for each observation can be identified. By using Shephard's input distance function to represent technology rather than the cost function, we can employ a dual Shephard's lemma to retrieve firm and input specific shadow prices.

2. The distance function approach

Again, let $L(u)$ denote the input set, then the Shephard input distance function may be defined as

$$\psi(u,x)=\sup\{\lambda>0\colon x/\lambda\in L(u)\}. \tag{2.1}$$

Clearly, $x\in L(u)$ if and only if $\psi(u,x)\geqq 1$. Moreover, given the cost function $C(u,p)=\min_x\{px\colon \psi(u,x)\geqq 1\}$, Shephard (1953, 1970) has shown that the input distance function may also be obtained as a price minimal cost function, i.e.

$$\psi(u,x)=\min_q\{qx\colon C(u,q)\geqq 1\}, \tag{2.2}$$

where in contrast to Shephard (1970, p. 276) we distinguish between prices p and cost normalized prices q.

For the moment, suppose that the distance function (2.2) is known, then by the dual Shephard's lemma, the optimal (cost deflated) shadow price vector $q(u,x)$ is also known, since

$$\nabla_x\psi(u,x)=q(u,x). \tag{2.3}$$

Next, we show how this price vector is related to the shadow price vector of the Atkinson and Halvorsen type. Thus, consider the primal cost minimization problem

$$\min_x\{p^{\mathrm{s}}x\colon \psi(u,x)\geqq 1\}.$$

From this problem we obtain

$$p^{\mathrm{s}}=C(u,p^{\mathrm{s}})\nabla_x\psi(u,x). \tag{2.4}$$

To show that (2.4) holds, consider the cost minimization problem as a Lagrangian problem

$$\Lambda = p^s x - \lambda(\psi(u,x) - 1).$$

The first-order conditions with respect to the inputs are

$$p^s = \lambda(u,x)\nabla_x\psi(u,x).$$

Following Jacobsen (1972) or Shephard (1970) one can show that $\lambda(u,x) = C(u,p)$ at the optimum.

Thus, when duality holds, by (2.3) and (2.4) we have

$$\frac{p^s}{C(u,p^s)} = q(u,x). \tag{2.5}$$

Suppose that there are $j=1,\dots,J$ observations of inputs $(x_1^j,\dots,x_N^j)$, outputs $(u_1^j,\dots,u_M^j)$ and input prices $(p_1^j,\dots,p_N^j)$, then the individual values of the k_n's, i.e. k_n^j, $j=1,\dots,J$, $n=1,\dots,N$ may be obtained from (2.5) under the assumption that $k_1^j=1$, $j=1,\dots,J$. That is,

$$k_n^j = \frac{p_1^j}{p_n^j}\frac{q_n(u^j,x^j)}{q_1(u^j,x^j)} = \frac{p_1^j}{p_n^j}\frac{p_n^{sj}}{p_1^{sj}}, \quad j=1,\dots,J,\ n=2,\dots,N. \tag{2.6}$$

Thus, when the distance function (2.2) is known, then (2.6) shows how individual values of k can be deduced, given the normalization $k_1^j=1$, $j=1,\dots,J$.

It remains to prove that a distance function can be estimated. The assumption that $L(u)$ is a closed convex set implies that the two approaches, (2.1) and (2.2), yield the same distance function [see Shephard (1953, 1970)]. From this observation it follows that $\psi(u,x)$ can be calculated using the formulation in (2.1) which only requires data on input and output quantities. If we parameterize ψ, as suggested by Diewert (1976), as a translog distance function, we may apply the parametric linear programming method introduced by Aigner and Chu (1968) to compute its parameters. Evaluation of the derivative for each observation with respect to the input vector yields $q_n(u^j,x^j)$, $n=1,\dots,N$, $j=1,\dots,J$.[2] These in turn may be used to calculate

[2]One may also identify the individual undeflated shadow prices, $p_n^{j'}$, $j=1,\dots,J$, $n=1,\dots,N$, if one is willing to make one of the following assumptions:

A.1. One input market is efficient, i.e. $p_{n'}^s = p_n^o$ for some n.

A.2. Firms satisfy a balanced budget or not for profit constraint.

With A.1, one may use the observed efficient input price, say $p_{n'}^o$ to deduce minimal costs since $q_{n'}^{jo} = p_{n'}^{jo}/C^j$; see Färe, Grosskopf and Nelson (forthcoming). With A.2, C^j may be retrieved since costs must equal revenues.

the individual values of the k_n^j's as in (2.6). This approach also yields individual estimates of technical efficiency of the Farrell (1957) type.

As an alternative, which is also a frontier approach, but which is stochastic, one may parameterize (2.1) as a stochastic frontier model. Again one may prefer a flexible parameterization such as the translog. The frontier distance function differs from the more familiar frontier production function[3] in that it readily allows for multiple outputs, and its frontier value is unity.[4]

The dual approach suggested here to identify firm and input specific shadow prices may be extended to identification of output shadow prices by modeling technology with a Shephard output distance function and applying the appropriate dual Shephard's lemma.

[3]See Lovell and Schmidt (1988) for an overview of frontier models.

[4]Since we are interested in identifying shadow prices which support technology, we would like to evaluate the derivatives along the surface or frontier of technology, i.e. where $\Psi(u,x)=1$. For the stochastic frontier translog model we would have:

$$\ln 1 = \alpha_0 + \sum_{m=1}^{M} \alpha_m \ln u_m + \sum_{n=1}^{N} \beta_n \ln x_n + \tfrac{1}{2} \sum_{m=1}^{M} \sum_{m'=1}^{M} \alpha_{mm'} \ln u_m \ln u_{m'}$$

$$+ \tfrac{1}{2} \sum_{n=1}^{N} \sum_{n'=1}^{N} \beta_{nn'} \ln x_n \ln x_{n'} + \sum_{m=1}^{M} \sum_{n=1}^{N} \gamma_{mn} \ln u_m \ln x_n + v,$$

where v is a composed error term, and appropriate restrictions to ensure homogeneity of degree plus one in inputs are imposed.

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