

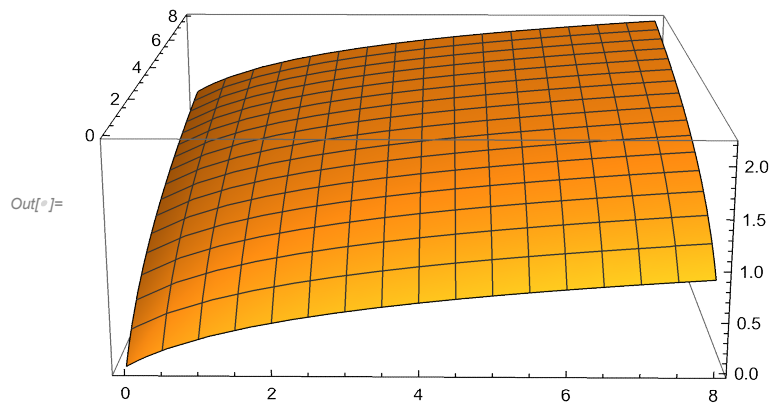
NB. La funzione è sempre U (W). In tutti gli esempi, x e y sono 2 realizzazioni di W date le prob.

Primo Esempio $U = \ln (W)$

$$U = 0.4 * \text{Log}[x + 1] + 0.6 * \text{Log}[y + 1]$$

$$\text{Out[*]} = 0.4 \text{Log}[1 + x] + 0.6 \text{Log}[1 + y]$$

$$\text{In[*]} = \text{Plot3D}[U, \{x, 0, 8\}, \{y, 0, 8\}]$$



$$\text{In[*]} = \mathbf{H} = \left\{ \left\{ \partial_{x,x}U, \partial_{x,y}U \right\}, \left\{ \partial_{y,x}U, \partial_{y,y}U \right\} \right\} // \text{MatrixForm}$$

Out[*]//MatrixForm=

$$\begin{pmatrix} -\frac{0.4}{(1+x)^2} & 0 \\ 0 & -\frac{0.6}{(1+y)^2} \end{pmatrix}$$

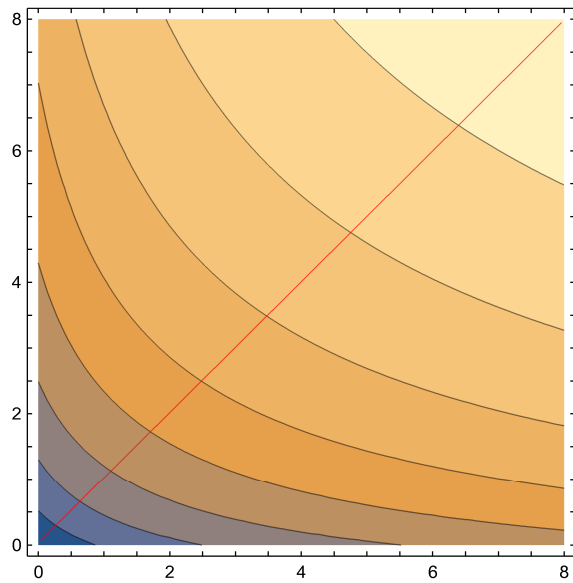
$$\text{In[*]} = \text{Eigenvalues}[\mathbf{H}]$$

$$\text{Out[*]} = \left\{ -\frac{0.6}{(1+y)^2}, -\frac{0.4}{(1+x)^2} \right\}$$

$$\text{In[*]} = \text{Det}[\mathbf{H}]$$

$$\text{Out[*]} = \frac{0.24}{(1+x)^2 (1+y)^2}$$

$$\text{In[*]} = \text{ContourPlot}[U, \{x, 0, 8\}, \{y, 0, 8\}]$$



$$\text{In[*]} := \text{SMS} = \frac{\partial_x U}{\partial_y U}$$

$$\text{Out[*]} = \frac{0.666667 (1 + y)}{1 + x}$$

$$\text{In[*]} := \partial_x \text{SMS}$$

$$\text{Out[*]} = -\frac{0.666667 (1 + y)}{(1 + x)^2}$$

$$\text{In[*]} := \mathbf{N} \left[\begin{array}{c} 0.4 \\ 0.6 \end{array} \right]$$

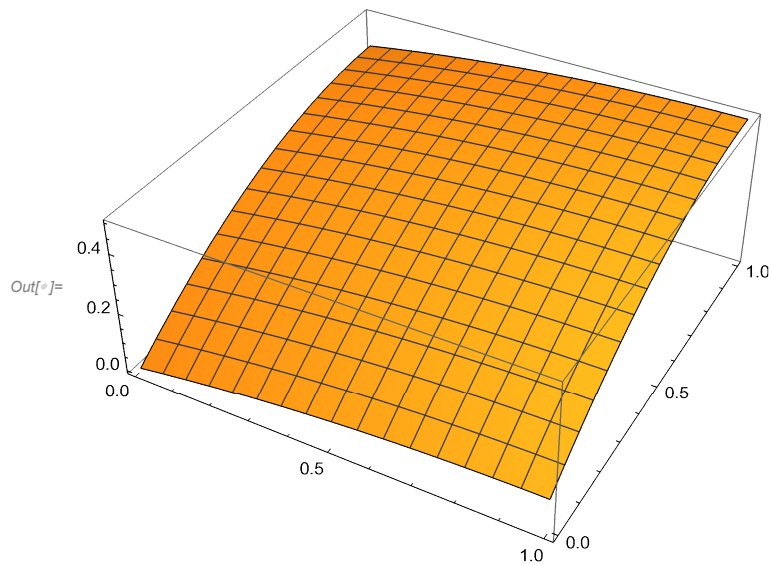
$$\text{Out[*]} = 0.666667$$

Secondo Esempio U quadratica

$$\text{In[*]} := U = 0.2 * \left(x - \frac{1}{2} x^2 \right) + 0.8 * \left(y - \frac{1}{2} y^2 \right)$$

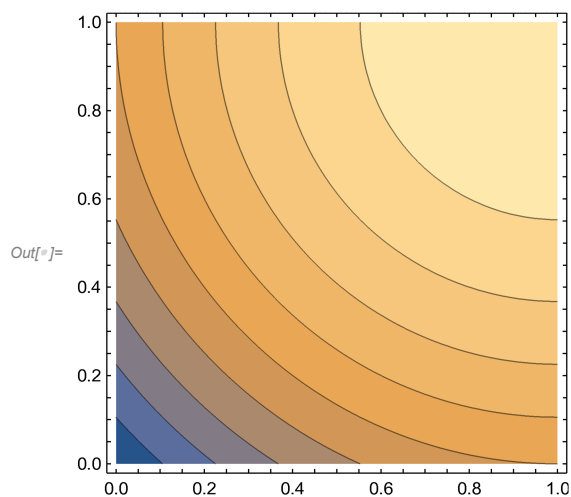
$$\text{Out[*]} = 0.2 \left(x - \frac{x^2}{2} \right) + 0.8 \left(y - \frac{y^2}{2} \right)$$

In[]:= Plot3D[U, {x, 0, 1}, {y, 0, 1}]



In[]:=

ContourPlot[$\left(x - \frac{1}{2}x^2\right) + \left(y - \frac{1}{2}y^2\right)$, {x, 0, 1}, {y, 0, 1}]



In[]:= H = {{∂_{x,x}U, ∂_{x,y}U}, {∂_{y,x}U, ∂_{y,y}U}} // MatrixForm

Out[]//MatrixForm=

$$\begin{pmatrix} -0.2 & 0 \\ 0 & -0.8 \end{pmatrix}$$

In[]:= Det[H]

Out[]:= 0.16

In[]:= NegativeSemidefiniteMatrixQ[H]

Out[]:= True

In[]:= SMS = $\frac{\partial_x U}{\partial_y U}$

Out[]:= $\frac{0.25(1-x)}{1-y}$

In[*]:= $N\left[\frac{0.2}{0.8}\right]$

Out[*]:= 0.25

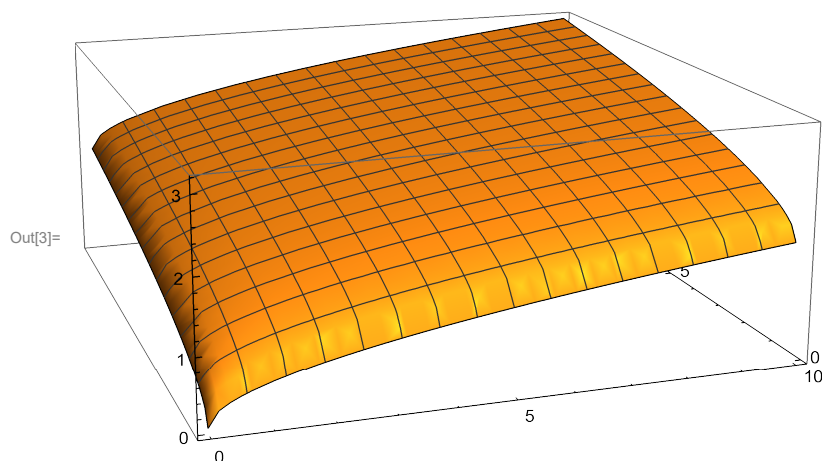
Terzo Esempio U come in Lagrange

$$U = \sqrt{w}$$

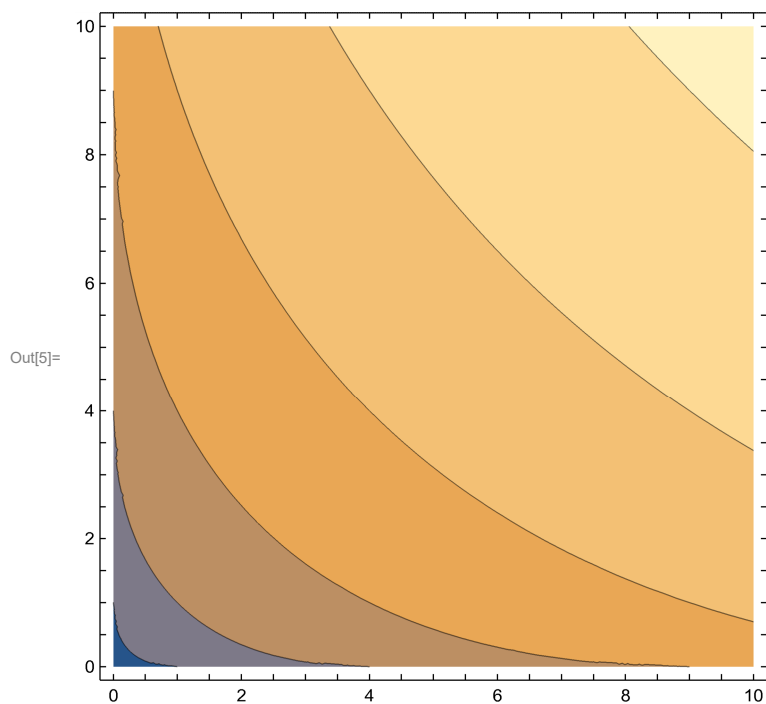
In[1]:= $U = 0.5 * \sqrt{x} + 0.5 * \sqrt{y}$

Out[1]:= $0.5 \sqrt{x} + 0.5 \sqrt{y}$

In[3]:= `Plot3D[U, {x, 0, 10}, {y, 0, 10}]`



In[5]:= `ContourPlot[U, {x, 0, 10}, {y, 0, 10}]`



In[8]:= $\mathbf{H} = \left\{ \left\{ \partial_{x,x} U, \partial_{x,y} U \right\}, \left\{ \partial_{y,x} U, \partial_{y,y} U \right\} \right\}$ // MatrixForm

Out[8]/MatrixForm=

$$\begin{pmatrix} -\frac{0.125}{x^{3/2}} & 0 \\ 0 & -\frac{0.125}{y^{3/2}} \end{pmatrix}$$

In[7]:= **Det [H]**

Out[7]= $\frac{0.015625}{x^{3/2} y^{3/2}}$

In[9]:= **SMS** = $\frac{\partial_x U}{\partial_y U}$

$$\frac{\sqrt{y}}{\sqrt{x}}$$

In[27]:= **Taylor P2 di U (W) quadratica**

Series [W - b * W², {W, μ, 2}]

Out[28]= $(\mu - b \mu^2) + (1 - 2 b \mu) (W - \mu) - b (W - \mu)^2 + O[W - \mu]^3$

In[26]:= **Normal [(μ - b μ²) + (1 - 2 b μ) (W - μ) - b (W - μ)² + O[W - μ]³]**

Out[26]= $-b (W - \mu)^2 + \mu - b \mu^2 + (W - \mu) (1 - 2 b \mu)$

Interpretare l' espansione e il polinomio ottenuto. Se μ è la media di W, scrivere il valore atteso del polinomio. Ricavare che la funzione valutata in valore atteso dipende solo da media (μ) e varianza.

Se cercassimo Series [W - b * W², {W, μ, 3}] cambierebbe qualcosa?