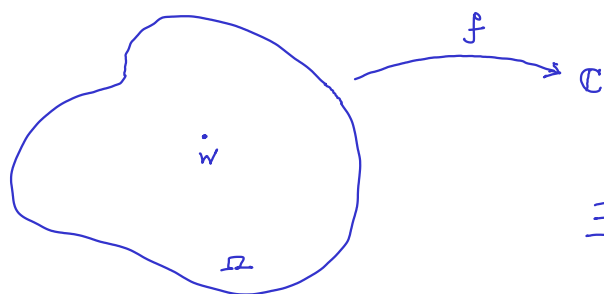


ANALISI COMPLESSA (I° SEM. - 6 CFU)



$$\exists \lim_{z \rightarrow w} \frac{f(z) - f(w)}{z - w} =: f'(z) \in \mathbb{C}$$

Risultati fotocopia di quelli visti in Analisi I per funzioni $g: I \rightarrow \mathbb{R}$ derivabili?

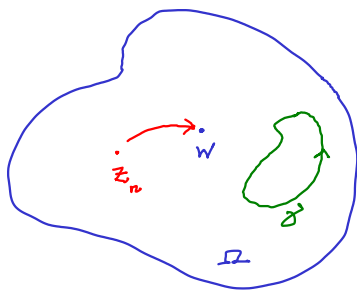
NO!!

FATTI "MAGICI"

f derivabile (in senso complesso)
una volta in Ω

\Rightarrow

- f derivabile infinite volte
- $f(z_n) = 0, n=1, 2, 3, \dots \Rightarrow f \equiv 0$
- $\int_{\gamma} f(dx + idy) = 0$



APPLICAZIONI?

NUMEROSE!

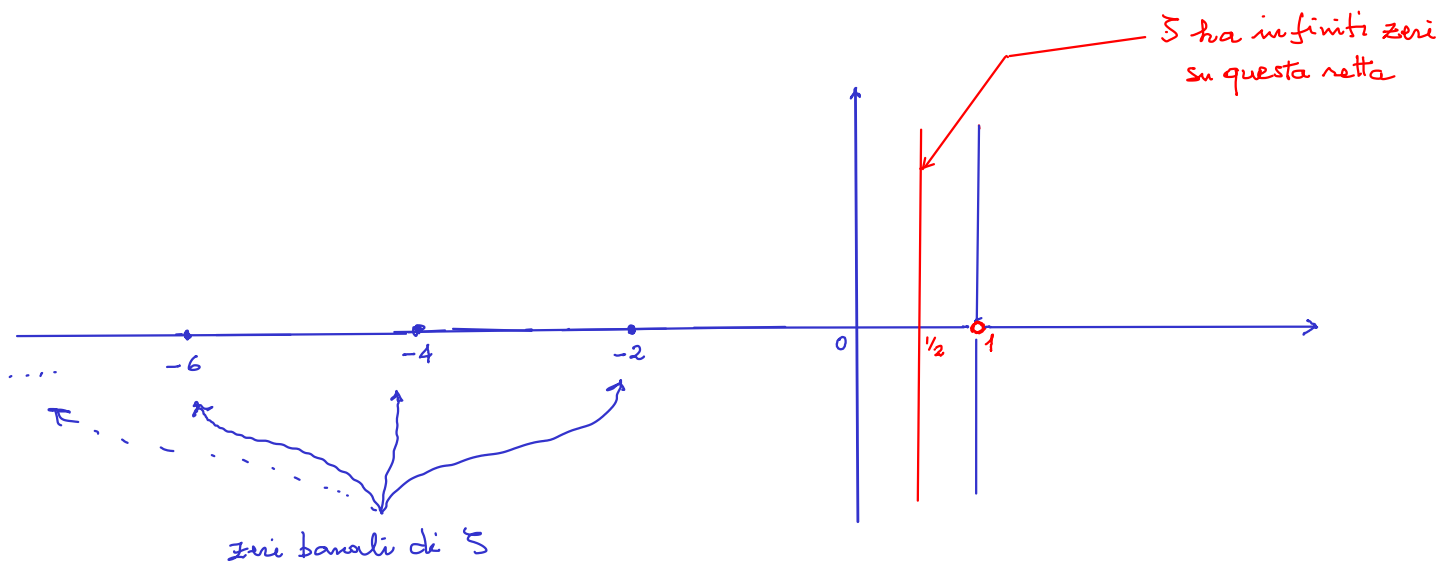
• $\pi(x) = \#\{p \text{ primo} : p \leq x\} \sim \frac{x}{\log x}$ (Hadamard, de la Vallée Poussin)

$= \frac{x}{\log x} + ??$

• $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$; $\sum_{n=1}^{\infty} \frac{1}{n^3} = ? \dots$

$\zeta(w) := \sum_{n=1}^{\infty} \frac{1}{n^w}$ funzione ζ di Eulero (Riemann)

PROBLEMA ζ ha zeri in $\{w : 0 \leq \text{Re } w \leq 1, \text{Re } w \neq 1/2\}$?



CONGETTURA (di Riemann) No, non ne ha !

E.M. Stein - R. Shakarchi,
Complex Analysis

