2D transforms for the odometry of a differential kinematics mobile robot

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Abstract

In this document we shortly present the the roto-translations involved in the update of the odometric pose estimate for a mobile robot. This task is performed in the planar (2D-3DoF) case and for a differential kinematics mobile robot. Therefore, we are just presenting a verbose description of how to obtain the well-known formulas for differential kinematics odometry. The development is here presented as an exercise in representation and composition of roto-translations with homogeneous coordinates, in 2D. We first introduce the problem, then develop the primitive roto-translations that make up the transformation between two consecutive readings of the wheel angular positions, and finally compose this transform with the previous pose.

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I. Introduction

Suppose to have a differential kinematic mobile robot, which is the easiest to build, and likely the most used configuration of all wheeled mobile robot configurations. For a short introduction and review of mobile robot configurations, see e.g.,the text of R. Siegwart [siegwart].

The robot will therefore have, for traction, a left and a right wheel. Of course, it will also have at least another contact point (e.g., a castor-wheel), but, as this is not significant for odometry, we'll simply avoid mentioning it. The robot will move along a trajectory that will be locally orthogonal to the line joining the contact points of the two wheels. This line will be mentioned hereafter as the wheel baseline of the robot. The length of the baseline is assumed known, the radius of the wheels is assumed known.

For what concerns this exercise, the robot will move in a planar environment; although this is indeed a questionable hypothesis, we are going to base on it.

We assume here that the reader has a clear understanding of the reasons for having a wheel-based odometric system for mobile robots, and also why this is meaningless for manipulators; in case refer again to the text by R. Siegwart.

Periodically, we read the angular position of the wheels, which we suppose are sporting a suitable sensor of the angular position, e.g., an incremental encoder. Therefore we have a discrete-time system, i.e., time is indexed with integers: at time t the robot is in pose R_t, while at the previous time (t_1) it was in R_{t-1}.

For each reading we know the angular orientation of each wheel with respect to the previous reading. We can therefore transform this angular motion into the length of the arcs that have been traversed by the contact point of each wheel, under the following hypotheses:

- known (and constant) radius of each wheel,
- no-slippage of wheel contact point,
- etc. (i.e., other questionable hypotheses concerning the contact between the wheel and the floor, which we take for granted for this exercise).

The transformation of the angular motion of the wheel into the length of the arc traversed by the contact point is performed for both wheels and is not considered in this exercise.

We also know the previous robot pose, so that the planar motion (roto-translation) that took place during the last time interval, mentioned hereafter as the *elementary* transform, can be composed with the previous robot pose to obtain the current robot pose.

We also hypothesize that the time interval between two consecutive readings of the position of the wheels to be so small that the motion taking place during the interval can be approximated to a constant velocity motion.

Then, at each time interval, we have a roto-translation, and this can be reduced to a pure rotation around a suitable pole (Mozzi + Chasles theorem for a planar motion), the so-called Centro di Istantanea Rotazione (CIR) or Instantaneous Center of Rotation or Instant Center of Rotation.

Therefore, the elementary transform can be obtained by the composition of three *primitive* transforms, refer to the sketch in Figure 1, $T_{R_t}^{CIR_t}$, $T_{CIR_t}^{CIR_{t-1}}$, $T_{CIR_{t-1}}^{R_{t-1}}$.

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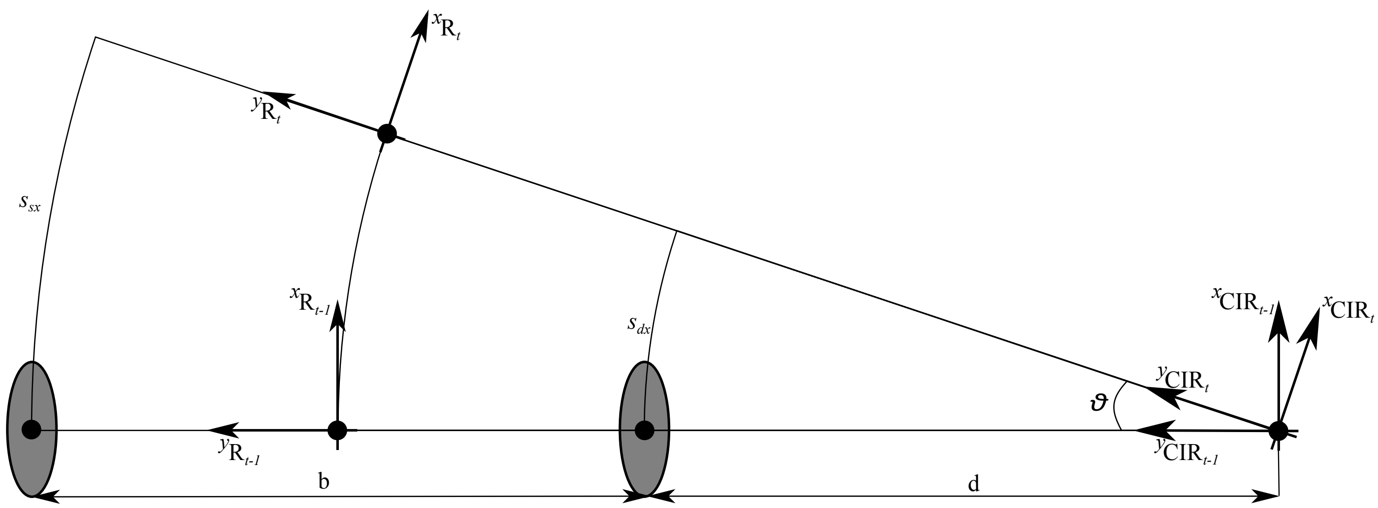

Fig. 1. Sketch of the problem; b=baseline, d=distance to CIR, s_{sx} = left arc, s_{dx} = right arc, R = robot pose.

The CIR is found at the intersection of the two baselines, one defined by the contact points of the wheels before the motion (robot pose = R_{t-1}), and the other one defined by the contact points of the wheels at the end of the motion (robot pose = R_t), see Figure 1.

The parameters of the elementary transform are d, the distance to the CIR, and ϑ, the angle of rotation about the CIR. If we know these two parameters, we can write down the primitive transforms, i.e., the components of the elementary transform.

II. Determination of the parameters of the elementary transform

To model the elementary transform $T_{R_t}^{R_{t-1}}$ we need to know the distance d of the CIR along the baseline, as well as the rotation angle ϑ about the axis orthogonal to the motion plane and passing through the CIR.

We can observe that:

$$\begin{cases} s_{dx} = d\,\vartheta \\ s_{sx} = (d+b)\,\vartheta \end{cases} ; \quad \begin{cases} d = s_{dx}/\vartheta \\ s_{sx} = (s_{dx}/\vartheta + b)\,\vartheta \end{cases} ; \quad \begin{cases} d = s_{dx}/\vartheta \\ s_{sx} = s_{dx} + b\,\vartheta \end{cases} ; \quad \begin{cases} d = s_{dx}/\vartheta \\ \vartheta = (s_{sx} - s_{dx})/b \end{cases} ; \quad \begin{cases} d = s_{dx}\; b/(s_{sx} - s_{dx}) \\ \vartheta = (s_{sx} - s_{dx})/b \end{cases}$$

III. The primitive transforms

The primitive transforms required to build up the elementary transform can now easily determined, changing the signs as required to the parameters determined above:

IV. The elementary transform

We can now compose the primitive transforms to build the elementary transform, by following the usual concatenation rules.

V. The odometric estimate of the robot pose

We can now pass to the last step, i.e., the composition of the elementary transform with the previous robot pose estimate, so to obtain an updated pose with respect to the world reference frame.

At time $t = 1$, supposing to have the world in coincidence with the robot frame at time $t = 0$, we have:

$$\mathbf{T}_{R_0}^{W} = \mathbf{I};$$

$$\mathbf{T}_{R_1}^{W} = \mathbf{T}_{R_1}^{R_0};$$

At a generic time t we have:

$$\mathbf{T}_{R_t}^{W} = \mathbf{T}_{R_{t-1}}^{W} \cdot \mathbf{T}_{R_t}^{R_{t-1}}$$

where $\mathbf{T}_{R_{t-1}}^{W}$ represents the pose of the robot (X, Y, α) at time $t-1$, in the form of a roto-translation matrix in homogeneous coordinates in the plane, with respect to the world reference frame.

After giving out the odometric estimate at the current time, i.e., concluding the current time iteration, the current estimate becomes the previous estimate for the forthcoming new reading of the two arcs (s_{sx} and s_{dx}), and then the procedure is iterated.