Un plasma è un gas che contiene una certa quantità di part. cariche, che domina la dinamica del sistema esibendo un comp. collettivo

Eq. Saha-Boltzmann

$$N \propto \exp\left(-E/T\right) \cdot g$$ degenerazione

atomo idrogeno GS

$$E = -Ry/n^2$$

$n = 1$ $\quad l = 0 \dots n-1$ $\quad l = 0$

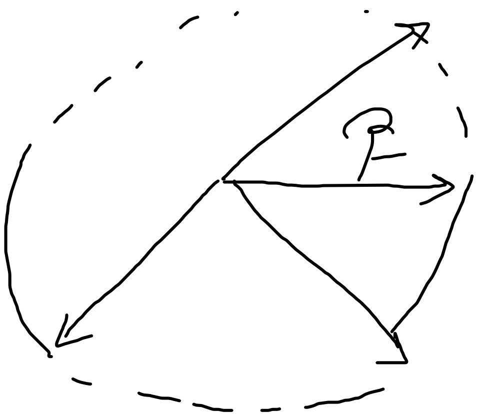
$m = -l \dots l$ $\quad m = 0$ $\quad s = \frac{1}{2}, -\frac{1}{2}$

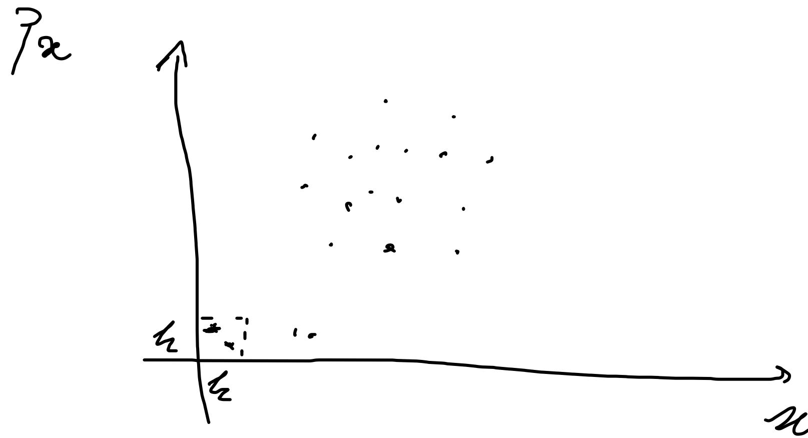
$$E = \chi + \frac{p^2}{2m}$$

en. interna ione

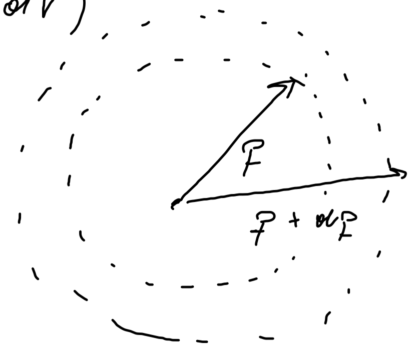
p: momento lineare

$$\Delta x \, \Delta p_x \geq \frac{\hbar}{2}$$





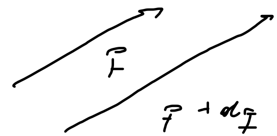
spazio disponibile per part. $(\vec{p}, dV)$ $= 4\pi p^2\, dp\, dV$



$\Delta x \Delta p_x \sim h$

$\Delta y \Delta p_y \sim h$

$\Delta z \Delta p_z \sim h$



$$dV \cdot (\Delta p)^3 \sim h^3$$

$$\#\text{stati} = \frac{4\pi p^2\, dp\, dV}{h^3}$$

$$\frac{4}{3}\pi (p+dp)^3 - \frac{4}{3}\pi p^3$$

$$= \frac{4}{3}\pi p^3 \left(1+\frac{dp}{p}\right)^3 - \frac{4}{3}\pi p^3$$

$$\simeq \frac{4}{3}\pi p^3 \left(\cancel{1}+ \frac{3dp}{p}\right) - \cancel{\frac{4}{3}\pi p^3} = 4\pi p^2\, dp$$

$$\frac{N_{ion}}{N_{neutre}} = \frac{g_i}{g_S} \cdot \frac{4\pi p^2 \, dp \, dV}{h^3} \frac{\exp\left(-\frac{E_i}{T}\right)}{\underbrace{\exp\left(-\frac{E_n}{T}\right)}}$$

$$\exp\left(-\frac{(E_i - E_n)}{T}\right) = \exp\left(\frac{-\chi - p^2/2m}{T}\right)$$

$$E_i = E_n + \chi + p^2/2m$$

pot. ioniz.

$n_e = \frac{\# \text{ elettroni}}{\text{volume}} \longrightarrow dV = \frac{1}{n_e}$

$$\frac{N_{ion}}{N_{neutre}} = g_i/g_S \int_0^{+\infty} \frac{4\pi p^2 \, dp \left(dV\right)}{h^3} \exp\left(\frac{-\chi - p^2/2m}{T}\right)$$

$d\frac{1}{n_e}$

$$J = \int_0^{+\infty} dp \; p^2 \exp\left(-\frac{p^2}{2mT}\right) = (2mT)^{\frac{3}{2}} \int_0^{+\infty} d\xi \; \xi^2 \exp(-\xi^2)$$

$$\xi \overset{def}{=} \frac{p}{\sqrt{2mT}} \qquad dp = \sqrt{2mT}\, d\xi \qquad \frac{d}{d\xi}\exp(-\xi^2) = -2\xi\exp(-\xi^2)$$

$$\xi^2 \cdot \exp(-\xi^2) = \xi \cdot \underline{\xi \exp(-\xi^2)}$$

$$= -\frac{\xi}{2}\frac{d}{d\xi}\exp(-\xi^2)$$

$$= -\frac{(2mT)^{\frac{3}{2}}}{2} \int_0^{+\infty} d\xi \; \xi \cdot \frac{d}{d\xi}\exp(-\xi^2) =$$

$$\nearrow f \qquad \nearrow g'$$

$$= -\frac{(2mT)^{\frac{3}{2}}}{2}\left[\cancel{\xi \cdot \exp(-\xi^2)\Big|_0^{+\infty}} - \int_0^{+\infty} \underbrace{\exp(-\xi^2)}_{\sqrt{\pi}/2} d\xi\right]$$

$$= \frac{(2mT)^{\frac{3}{2}}}{4} \cdot \sqrt{\pi}$$

$k_B \cdot T =$

$1.38 \cdot 10^{-23}\ J/K$

$T \sim 300\ K$

$k_B T \sim 25\ meV$

$$\frac{1.38 \cdot 10^{-23} \cdot 300}{1.6 \cdot 10^{-19}} =$$

$k_B T = 1\ eV \Rightarrow T \sim 12000\ K$

$$\frac{n_i}{n_n} = \underbrace{\frac{g_i}{g_n}}_{O(1)} \frac{2(2\pi m T)^{3/2}}{n_e h^3} \exp\left(-\chi_i/T\right) \quad ; \quad n_e \sim n_i$$

$$\frac{n_e}{n_n} \approx \left(\frac{g_i}{g_n} \frac{2(2\pi m T)^{\frac{3}{2}}}{n_n h^3} \exp\left(-\frac{\chi}{T}\right) \right)^{\frac{1}{2}}$$

n_n : legge gas ideali $\quad pV = n_{mol} R \cdot T = N_{part} \cdot k_B T$

$$n_n = \frac{N}{V} = \frac{p}{k_B T} \simeq 7 \cdot 10^{27} \frac{p[atm]}{T[K]} \; m^{-3}$$

$p = 1\,atm \quad T = 300\,K \qquad n_n \simeq 2 \cdot 10^{25}\, m^{-3}$

$T \sim 300\ K$ $\chi \sim 14.5\ eV$

N_2

$\frac{n_e}{n_n} \sim 10^{-122}$

$T \sim 12000\ K$ $k_B T \sim eV$

$\frac{n_e}{n_n} \sim 6\%$

$T \sim 10^6\ K \rightarrow k_B T \sim keV$

$\frac{n_e}{n_n} \sim 100\%$

gas