Temperatura

$$f(\underline{v}) = N \cdot \exp\left(-\frac{mv^2}{2k_BT}\right) \qquad d^3\underline{v} = dv_x\, dv_y\, dv_z$$

Richiesta:

$$\int f(\underline{v})\, d^3\underline{v} = n \qquad [n] = m^{-3}$$

$$[f] = m^{-3} / (m/s)^3$$

$$N \cdot \int \exp\left(-\frac{m(v_x^2+v_y^2+v_z^2)}{2T}\right) dv_x\, dv_y\, dv_z = N \cdot \int_{-\infty}^{+\infty} \exp\left(\frac{-mv_x^2}{2T}\right) dv_x \cdot \int_{-\infty}^{+\infty} \exp\left(\frac{-mv_y^2}{2T}\right) dv_y \cdot \int_{-\infty}^{+\infty} \exp\left(-\frac{mv_z^2}{2T}\right) dv_z$$

$$J = \int_{-\infty}^{+\infty} \exp\left(-\frac{mv_x^2}{2T}\right) dv_x \qquad \xi = \sqrt{\frac{m}{2T}}\, v_x \Rightarrow dv_x = \sqrt{\frac{2T}{m}}\, d\xi$$

$$J = \sqrt{\frac{2T}{m}} \underbrace{\int_{-\infty}^{+\infty} \exp(-\xi^2)\, d\xi}_{\sqrt{\pi}} = \left(\frac{2\pi T}{m}\right)^{\frac{1}{2}}$$

$$N \cdot \left(\frac{2\pi T}{m}\right)^{3/2} = n \rightarrow N = n \cdot \left(\frac{m}{2\pi T}\right)^{3/2}$$

$$f(\underline{v}) = n \left(\frac{m}{2\pi T}\right)^{3/2} \exp\left(-\frac{mv^2}{2T}\right)$$

$$\langle E \rangle = \frac{\sum_{\text{Tutte le } E} E \cdot \#\text{part. a quell'energia}}{\#\text{tot. particelle}} = \frac{\int \frac{1}{2} m v^2 f(\underline{v})\, d^3\underline{v}}{\int f(\underline{v})\, d^3\underline{v} = n}$$

$$\int \frac{1}{2} m v^2 \exp\left(-\frac{m v^2}{2T}\right) d^3v = \frac{1}{2} m \int \left(v_x^2 + v_y^2 + v_z^2\right) \cdot \exp\left(-\frac{m}{2T}\left(v_x^2 + v_y^2 + v_z^2\right)\right) dv_x\, dv_y\, dv_z =$$

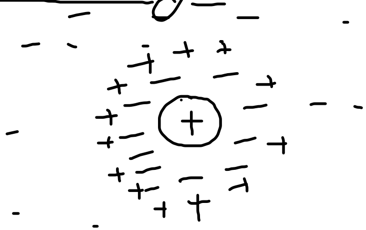
$$= \frac{1}{2} m \left[\int_{-\infty}^{+\infty} v_x^2 \exp\left(-\frac{m v_x^2}{2T}\right) dv_x \cdot \int_{-\infty}^{+\infty} \exp\left(-\frac{m v_y^2}{2T}\right) dv_y \cdot \int_{-\infty}^{+\infty} \exp\left(-\frac{m v_z^2}{2T}\right) dv_z + (\text{same}) + (\text{same}) \right]$$

$$\int_{-\infty}^{+\infty} v_x^2 \exp\left(-\frac{m v_x^2}{2T}\right) dv_x = \left(\frac{2T}{m}\right)^{\frac{3}{2}} \cdot \overbrace{\int_{-\infty}^{+\infty} \xi^2 \exp(-\xi^2)\, d\xi}^{\sqrt{\pi}/2} = \left(\frac{2T}{m}\right)^{\frac{3}{2}} \cdot \sqrt{\pi}/2$$

$$\xi = \sqrt{\frac{m}{2T}}\, v_x \rightarrow dv_x = \left(\frac{2T}{m}\right)^{\frac{1}{2}} d\xi$$

$$\langle E \rangle = \frac{3}{2} k_B T$$

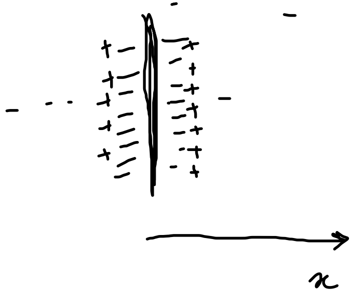
Schermatura di Debye



In vuoto: $\phi(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

In plasma?

$\phi_0 > 0, \sigma$



In vuoto:

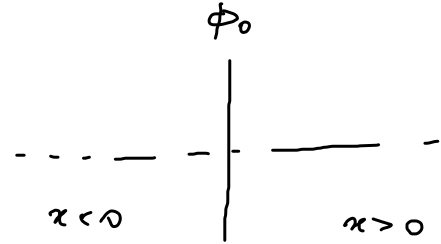
$\underline{\nabla}\cdot\underline{E} = \rho/\varepsilon_0$ $\quad\quad$ $\underline{E} = -\underline{\nabla}\phi$

In $x > 0$, $x < 0$ $\quad$ $\underline{\nabla}\cdot\underline{E} = 0$ $\quad\quad$ $\rho = \sigma\cdot\delta(x)$

$\nabla^2\phi = 0$ $\quad\quad$ $\underline{\nabla}\cdot\underline{E} = \frac{\sigma\,\delta(x)}{\varepsilon_0}$

$\nabla^2\phi = -\frac{\sigma\,\delta(x)}{\varepsilon_0}$

$$\frac{d^2\phi}{dx^2} = 0 \ ; \qquad \frac{d\phi}{dx} = A \ ; \qquad \phi(x) = A\cdot x + B$$



$x>0$

$x<0$

$$\phi(x) = \begin{cases} A_1 x + B_1 & x>0 \\ A_2 x + B_2 & x<0 \end{cases}$$

$\phi(x=0) = \phi_0$ (cond. contorno)

$\phi(x)$ continua

$$\lim_{x\to 0^-} \phi(x) = \lim_{x\to 0^+} \phi(x) = \phi_0$$

$$B_2 = B_1 = \phi_0$$

$$\phi(x) = \phi(-x)$$

$$A_1 x + \cancel{\phi_0} = -A_2 x + \cancel{\phi_0}$$

$$A_1 = -A_2$$

$$\frac{d^2\phi}{dx^2} = -\frac{\sigma}{\varepsilon_0}\delta(x)$$

Trucco:

$$\lim_{\varepsilon\to 0}\int_{-\varepsilon}^{\varepsilon}\frac{d^2\phi}{dx^2}\,dx = -\frac{\sigma}{\varepsilon_0}\underbrace{\int_{-\varepsilon}^{\varepsilon}\delta(x)\,dx}_{1} = -\frac{\sigma}{\varepsilon_0}$$

$$\left.\frac{d\phi}{dx}\right|_{0^+} - \left.\frac{d\phi}{dx}\right|_{0^-} = -\frac{\sigma}{\varepsilon_0}$$

$$A_1 - A_2 = -\frac{\sigma}{\varepsilon_0}; \qquad 2A_1 = -\frac{\sigma}{\varepsilon_0}; \quad A_1 = -\frac{\sigma}{2\varepsilon_0}$$

$$\phi(x) = \begin{cases} -\frac{\sigma}{2\varepsilon_0}x + \phi_0 & x>0 \\ \frac{\sigma}{2\varepsilon_0}x + \phi_0 & x<0 \end{cases} \qquad E = -\frac{d\phi}{dx} = \begin{cases} \frac{\sigma}{2\varepsilon_0} & x>0 \\ -\sigma/2\varepsilon_0 & x<0 \end{cases}$$

$$\nabla^2 \phi = -\frac{\sigma}{2\varepsilon_0}\delta(x) + \rho_{pl}(x)/\varepsilon_0 \qquad \rho(x) = e n_i(x) - e n_e(x)$$

$$f(\underline{v}) = N \exp\left(\frac{-mv^2}{2T}\right)$$

$$\longrightarrow N \cdot \exp\left(-\frac{\left(\frac{1}{2}mv^2 + q\phi(x)\right)}{T}\right)$$

$$n(x) = \int d^3\underline{v}\, f(\underline{v}) = \underbrace{\int d^3\underline{v}\, N \exp\left(\frac{-mv^2}{2T}\right)}_{n_{imp}} \cdot \exp\left(-\frac{q\phi(x)}{T}\right) = n_{imp} \cdot \exp\left(-\frac{q\phi(x)}{T}\right)$$

In $x>0$, $x<0$ $\quad \frac{d^2\phi}{dx^2} = -\rho/\varepsilon_0 = -\frac{e n_{imp}}{\varepsilon_0}\left(1 - \exp\left(\frac{+e\phi}{Te}\right)\right) \qquad \frac{e\phi}{Te} \ll 1$

$$\exp\left(\frac{e\phi}{T_e}\right) \approx 1 + \frac{e\phi}{T_e}$$

$$\frac{d^2\phi}{dx^2} = \frac{e^2 n_{imp}}{T_e \varepsilon_0}\phi$$

Lungh. Debye

$$\frac{1}{\lambda^2_{De}} \stackrel{def}{=} \frac{e^2 n_{imp}}{\varepsilon_0 T_e}$$

$$\frac{d^2\phi}{dx^2} = \phi/\lambda^2_{De}$$

$$\phi(x) = \phi_0 e^{-|x|/\lambda_D}$$

$$\phi(x) = Ae^{x/\lambda_D} + Be^{-x/\lambda_D}$$

$A_1 = 0$

$\phi(x)$ continuo

$$\phi(x) = \begin{cases} A_1 e^{x/\lambda_D} + B_1 e^{-x/\lambda_D} & x>0 \\ A_2 e^{x/\lambda_D} + B_2 e^{-x/\lambda_D} & x<0 \end{cases}$$

$B_2 = 0$

$$\lim_{x\to 0^-} \phi(x) = \lim_{x\to 0^+} \phi(x)$$

$$A_2 = B_1 = \phi_0$$

$$E(x) = -\frac{d\phi}{dx} = \begin{cases} \frac{\phi_0}{\lambda_D} e^{-x/\lambda_D} & x > 0 \\ -\frac{\phi_0}{\lambda_D} e^{x/\lambda_D} & x < 0 \end{cases}$$

$$\left.\frac{d\phi}{dx}\right|_{0^+} - \left.\frac{d\phi}{dx}\right|_{0^-} = -\frac{\sigma}{\varepsilon_0} \qquad -\frac{\phi_0}{\lambda_D} - \frac{\phi_0}{\lambda_D} = -\frac{\sigma}{\varepsilon_0}$$

$$\Rightarrow \frac{\phi_0}{\lambda_D} = \frac{\sigma}{2\varepsilon_0}$$

$$|E| = \frac{\sigma}{2\varepsilon_0} e^{-|x|/\lambda_D}$$

$$\lambda_{De}^2 = \frac{\varepsilon T}{n e^2}$$

Criterio:

$$\frac{4}{3}\pi\lambda_D^3 \cdot n \gg 1$$

$$L \gg \lambda_D$$

dimensioni plasma

$$\lambda_D \simeq 69 \cdot \left(\frac{T}{n}\right)^{\frac{1}{2}} m \quad [T] \text{ in } K, \ [n] \text{ in } m^{-3}$$

Plasma fusione nucleare

$n \sim 10^{20}\ m^{-3}$ $\quad$ ($gas\ stp \sim 10^{25}\ m^{-3}$)

$T \sim 10^{7}\ K$

$\lambda_D \approx 20\ \mu m$ $\quad$ $N_{Debye} \sim 4 \cdot 10^{6}$

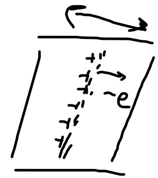
$L \sim m$

$\tau \gg \omega^{-1}$

tempo tra collisioni

freq. plasma

$$\omega_{Pe}^2 = \frac{n e^2}{m_e \varepsilon_0}$$



$F = eE$

$a = \frac{eE}{m}$

$\omega = 2\pi\nu \Rightarrow \nu = 9\sqrt{n_e}$ Hz $\qquad [n_e] = m^{-3}$

$\nu_p \sim 90$ GHz

Plasma: "buon" grado di ionizzazione (eq. Saha - Boltz)

T $\quad n_{ion}$ $\qquad \frac{4\pi\lambda_D^3}{3} \gg 1$ $\qquad L \gg \lambda_D$

$\tau \gg \omega^{-1}$

Moto delle cariche in $\underline{E}$ e $\underline{B}$

$\underline{B}$ uniforme

$$\underline{v} \cdot \; m\frac{d\underline{v}}{dt} = \underbrace{q(\underline{v}\times\underline{B})\cdot\underline{v}}$$

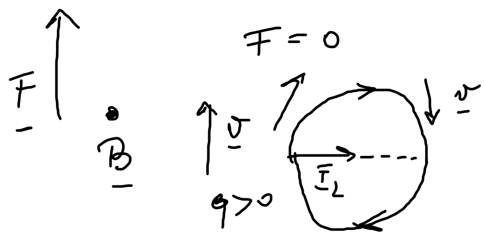
$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = 0$$

Lungo il campo
moto rett. unif.
$v_{\parallel}$

$\perp$ a $\underline{B}$ moto circolare $\quad r_L = \frac{m v_\perp}{qB} \quad \omega_L = qB/m \quad m\frac{d\underline{v}}{dt} = q(\underline{v}\times\underline{B}) + \underline{F}$

aggiungo $\underline{F} = \vec{\text{cost}}$

$F = 0$



$F \neq 0$

$\underline{v} = \underline{v}_L + \underline{v}_{\text{deriva}}$