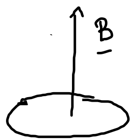
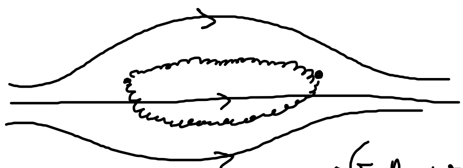
Se q é periodica : $J = \oint p \, dq$ é inv. adiabatico

$$p = \frac{\partial \mathcal{L}}{\partial \dot{q}}$$



$\Rightarrow \mu$ é inv. adiabatico



Moto periodico di "rimbalzo" tra i 2 pti di inversione del moto

Corrisponde

$$J = \int p_\parallel \, ds$$

$$p_\parallel = m v_\parallel + q A_\parallel$$

$$v^2 = v_\parallel^2 + v_\perp^2 \qquad q\left(v_\parallel A_\parallel + v_\perp A_\perp\right)$$

$$\mathcal{L} = \frac{1}{2} m v^2 + q \underline{v} \cdot \underline{A} - q \phi(\underline{x}, t)$$

$$\frac{\partial \mathcal{L}}{\partial v_\parallel} = m v_\parallel + q A_\parallel$$

$$\left[\underline{\nabla} \times \underline{B} = \mu_0 \underline{j} \right]_{\parallel} \qquad \underline{\text{Th}} \text{ ampere}$$

$$\left[\underline{\nabla} \times \left[\underline{\nabla} \times \underline{A} \right] \right]_{\parallel} = \mu_0 j_{\parallel}$$

di può scegliere

$$\cancel{\underline{\nabla} \underbrace{\left[\underline{\nabla} \cdot \underline{A} \right]}_{\text{cm}}}_{\parallel} - \nabla^2 \underline{A}_{\parallel} = \mu_0 j_{\parallel} \quad ; \quad \nabla^2 \underline{A}_{\parallel} = -\mu_0 j_{\parallel}$$

Se $j_{\parallel} = 0$ allora $A_{\parallel} = 0$

Gauge di Coulomb: $\underline{\nabla} \cdot \underline{A} = 0$

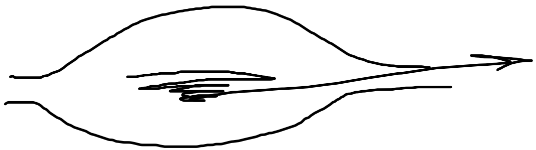
$$J = \int_{\text{orbita}} m v_{\parallel} \, d\underline{s} \sim m v_{\parallel} \cdot L$$

$L \downarrow \quad J = \text{const}$

$v_{\parallel} \uparrow$

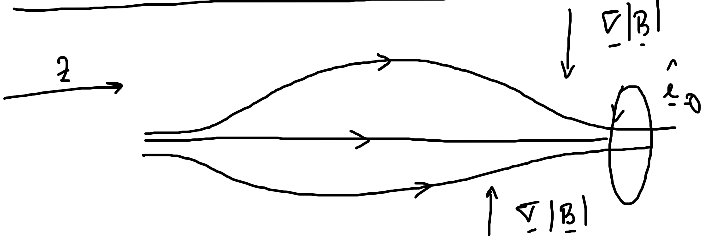
$$\operatorname{sen}\theta_{cr} = \left(\frac{v_{\perp 0}}{v_0}\right)^{\frac{1}{2}}$$

se I aumenta $\rightarrow$ $v_\parallel \uparrow$, $\operatorname{sen}\theta_{cr} \downarrow$ $\Big\}$ particella carica può uscire dal sistema



Meccanismo di acc. di Fermi

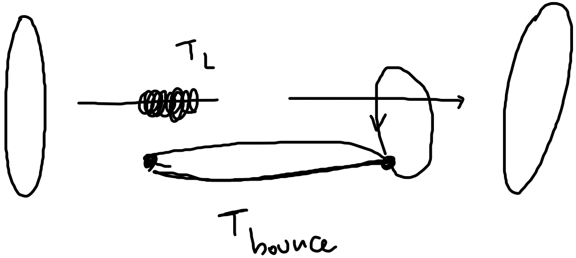
Terzo invariante adiabatico



$$\underline{v}_D = \frac{\underline{F} \times \underline{B}}{qB^2} \qquad \underline{F} \text{ a } \underline{\nabla}|\underline{B}| \propto \hat{\underline{e}}_r$$

$$\underline{B} \parallel \hat{\underline{z}}$$

$$\hat{\underline{e}}_r \times \hat{\underline{z}} = \hat{\underline{e}}_\theta$$



$T_{bounce} >> T_L$

Alla rotazione del pto di inversione del moto é associato

$$J = \int p\, dq \approx \int \left[m \langle \underline{v} \rangle + q \underline{A} \right] \cdot d\underline{r}$$

$d\underline{r}$: elemento infinitesimo di traiettoria del pto di inv. del moto

$J \Rightarrow$ flusso del $\underline{B}$ racchiuso nell'orbita seguita dai pti di inv. del moto é costante

Perché un inv. adiabatico sia tale serve che

$$\Delta t_{\text{variazione}} \gg T_{\text{periodico}}$$

$$T_{\text{Larmor}} < T_{\text{bounce}} < T_{\text{rotazione dei pti inv. moto}}$$

$$J_1 \qquad J_2 \qquad J_3$$

Interazione onde - carica (plasma)

Se $\begin{cases} \underline{E}(\underline{x},t) \\ \underline{B}(\underline{x},t) \end{cases}$ sono noti, "sappiamo" come si muovono le cariche

$\underline{x}(t) \qquad \underline{v}(t)$

Posso determinare $\underline{j} = \sum_{part.} n_p q_p \underline{v}_p \qquad \rho = \sum_{part} n_p q_p$

$\underline{\nabla} \cdot \underline{E} = \rho/\varepsilon_0 \qquad \underline{\nabla} \times \underline{E} = -\partial \underline{B}/\partial t$

$\underline{\nabla} \cdot \underline{B} = 0 \qquad \underline{\nabla} \times \underline{B} = \mu_0 \underline{j} + \varepsilon_0 \mu_0 \partial \underline{E}/\partial t$

$n_{part} \sim 10^{20}\ m^{-3}$

$L \sim m$

$\Rightarrow N = 10^{20}$

Valuto traiettore

Note $(\rho, \underline{j})$ valuto $\underline{E}, \underline{B}$

1) Approccio statistico

$$f(\underline{x}, \underline{v}, t)$$

eq. Vlasov (Fisica dei Plasmi 1)

2) Q.tà medie

$$\underline{u}(\underline{x}, t)$$
$$n(\underline{x}, t)$$

~~Risonanze~~

Plasma da fusione

$n \sim 10^{20} \, m^{-3}$ $\Delta x \sim 10^{-4} \, m$

$L \sim m$

$(\Delta x)^3 \sim 10^{-12} \, m^3$

$N = n \cdot (\Delta x)^3 \sim 10^8$

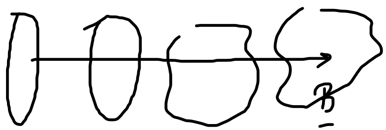
Plasma "astrofisico"

vento solare $\quad n \sim 5 \cdot 10^{6}\ m^{-3}$

$L \sim$ qualche R_{sun} $\qquad R_{sun} \sim 7 \cdot 10^{8}\ m$

$\Delta x \sim m \qquad N \sim n(\Delta x)^{3} \sim 10^{18}$

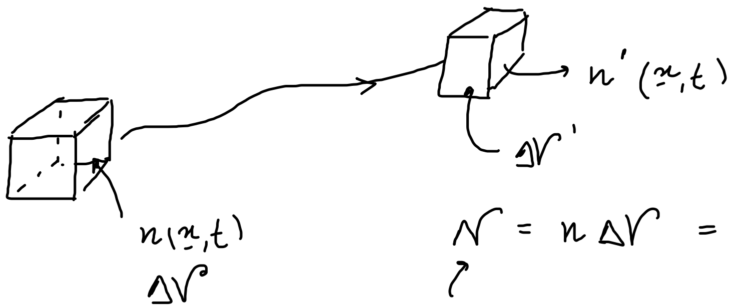
Fluido



Descrizione fluida debole lungo il campo magnetico

part. con vel. molto diversa dalle v_{th}

Conserv. del # particelle in un elemento di volume



$N = n\,\Delta V$ = cost. nel tempo

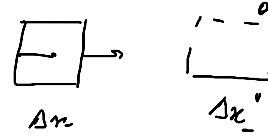
\# part. nel volume

Richiesta $\dfrac{dN}{dt} = 0$

$$\frac{d}{dt}(n\,\Delta V) = \frac{d n(\underline{x},t)}{dt}\cdot \Delta V + \frac{d}{dt}(\Delta V)\cdot n$$

$$\frac{d}{dt}(\Delta x\,\Delta y\,\Delta z) = \frac{d}{dt}(\Delta x)\,\Delta y\,\Delta z + \frac{d}{dt}(\Delta y)\,\Delta x\,\Delta z + \frac{d}{dt}(\Delta z)\,\Delta x\,\Delta y$$

$\dfrac{d}{dt}\Delta x =$



$= \lim_{\delta t \to 0}\left[\dfrac{\Delta x' - \Delta x}{\delta t}\right]$