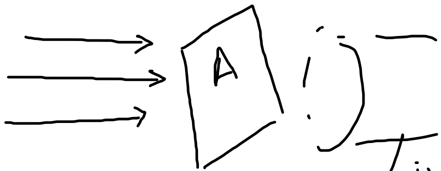
Flusso di un vettore attraverso una sup.

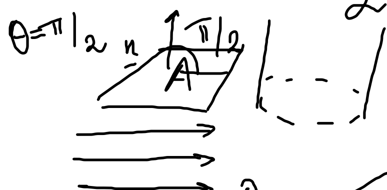


$\underline{E} \perp$ sup. A

$[\Phi] = \frac{N}{C} \cdot m^2$

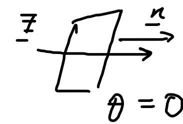
$\Phi = E \cdot A$

Linee di campo: $\#$ linee campo / sup. $\propto E$

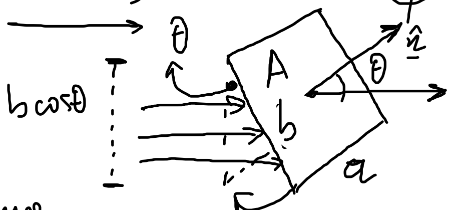


$\theta = \pi/2$

$\Phi = 0$



$\theta = 0$



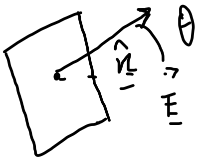
$\hat{\underline{n}}$: versore normale $\perp$ superficie, uscente

$\Phi \overset{def}{=} E \underbrace{A_{\cos\theta}}_{A \text{ proiettata}}$ nella dir. $\perp$ a $\underline{E}$

Supponiamo $A = a b$

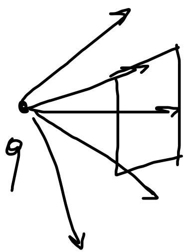
$A_{\cos\theta} = a \cdot b\cos\theta = A_{\perp}$

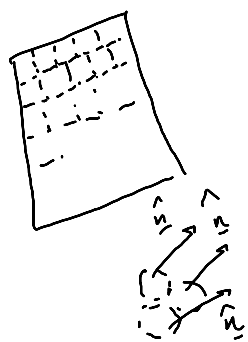
Vettore area



$$\underline{A} \overset{def}{=} \begin{cases} \text{modulo: } A \\ \text{direzione e verso: } \hat{\underline{n}} \end{cases} \qquad \underline{A} \overset{def}{=} A\,\hat{\underline{n}}$$

$$\Phi = EA\cos\theta = \underline{E}\cdot\underline{A}$$





Divido A (macroscopica) in tanti piccoli (infinitesimi) elementi di area dA

$d\underline{A}$

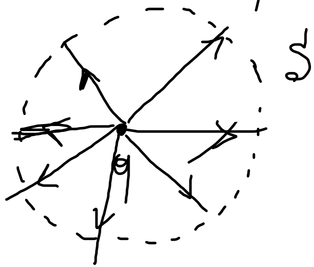
$$\Phi_{dA} = \underline{E}\cdot d\underline{A}$$

$$\Phi = \sum_{\text{tutta } A} \underline{E}\cdot d\underline{A} = \int_{\text{area}} \underline{E}\cdot d\underline{A}$$

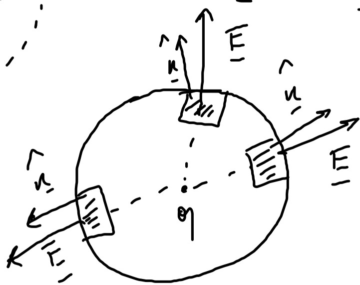
Teorema di Gauss

$\underline{E}$ di una carica puntiforme

Scelta di una sup. sf. di raggio r per calcolare il $\phi(\underline{E})$



$$\underline{E} = K_e \frac{q}{r^2}\hat{\underline{r}} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\underline{r}}$$



$\hat{\underline{r}}$ lungo il raggio per ogni elemento $d\underline{A}$

$\hat{\underline{n}} \parallel \underline{E}$ per ogni elemento $d\underline{A}$

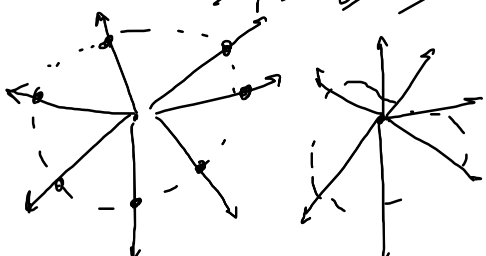
$$\Phi = \int_{sfera} \underline{E}\cdot d\underline{A} = \int_{sfera} \underbrace{\frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}}_{\underline{E}}\,\hat{r}\cdot\hat{n}\, dA = \frac{q}{4\pi\varepsilon_0}\int_{sfera}\frac{1}{r^2}\cdot dA = \frac{q}{4\pi\varepsilon_0 r^2}\underbrace{\int_{sfera} dA}_{\text{sup. sfera}}$$

$\hat{r} \parallel \hat{n}$: $\hat{r}\cdot\hat{n} = 1 = \cos(0^\circ)$

$$= \frac{q}{\cancel{4\pi\varepsilon_0 r^2}}\,\cancel{4\pi r^2} = \frac{q}{\varepsilon_0}$$

$$\oint_{sfera} \underline{E}\cdot d\underline{S} = \frac{q}{\varepsilon_0}$$ 1 carica puntiforme

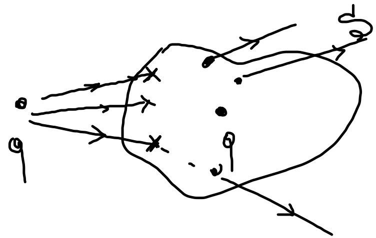
~~Sup. sferica~~



$$\oint_{\text{Sup. generica}} \underline{E}\cdot d\underline{S} = \frac{q}{\varepsilon_0}$$ 1 carica puntiforme

Sup. generica ~~centrata in q~~ contiene q

Più di una carica?



q interna: $\oint_{S \text{ generica}} \underline{E} \cdot d\underline{S} = q/\varepsilon_0$

q esterna: $\oint_{S \text{ generica}} \underline{E} \cdot d\underline{S} = 0$

$q_1 \dots q_N$ interne a S

$$\oint_S \underline{E} \cdot d\underline{S} \underset{\uparrow}{=} \oint_S \left(\sum_{i=1}^{N} \underline{E}_i \right) \cdot d\underline{S} = \oint_S \sum_{i=1}^{N} \left(\underline{E}_i \cdot d\underline{S} \right) = \sum_{i=1}^{N} \overbrace{\oint_S \underline{E}_i \cdot d\underline{S}}^{q_i/\varepsilon_0} = \sum_{i=1}^{N} \frac{q_i}{\varepsilon_0}$$

pr. sovr.

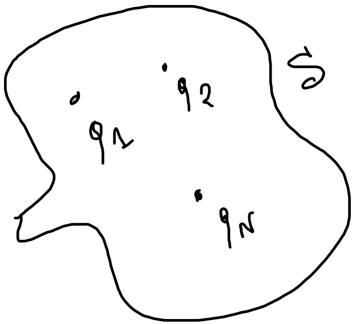
$\underline{E}_i$: c. elettrico prodotto da q_i

$= \frac{q^{int}}{\varepsilon_0}$

Teorema Gauss per l'elettrostatica

S è generica (geometrica) chiusa

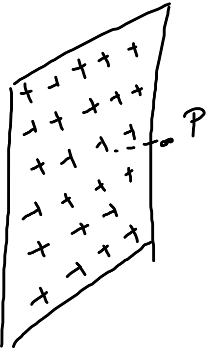
$$\oint_S \underline{E} \cdot d\underline{A} = \frac{q^{int}}{\varepsilon_0}$$

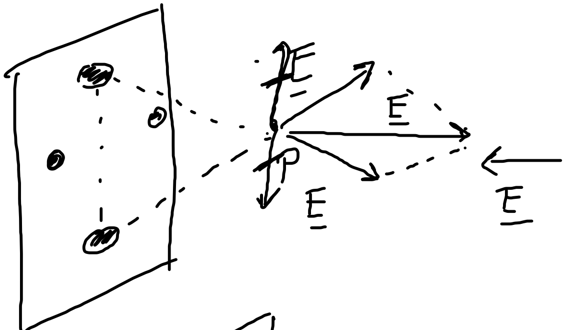


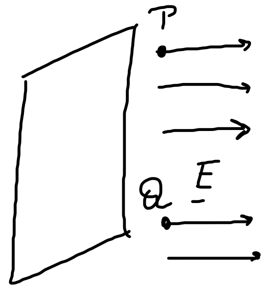
Se $\begin{cases} S \text{ è sufficient. regolare} \\ \text{il sistema è suff. simmetrico} \end{cases}$

allora potrebbe essere semplice risolvere ad $\underline{E}$ noto $\phi(\underline{E})$

Piano conduttore
unif. carico
molto esteso ("infinito")

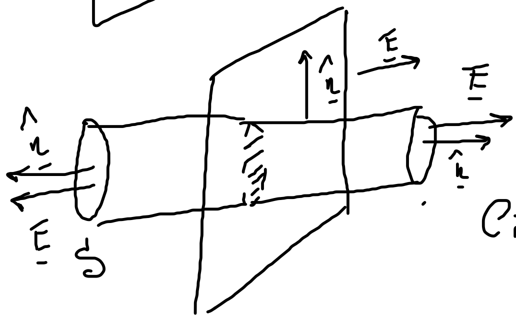






Linee di campo $\perp$ al piano

$\underline{E}$ non cambia in direz. spostandosi lungo il piano



S: cilindro tagliato dal piano

Cilindro: 3 sup. → 2 basi
→ sup. laterale

$$\oint_{\text{Cilindro}} \underline{E} \cdot d\underline{S} = \overbrace{\int_{\substack{\text{base}\\1}} \underline{E} \cdot d\underline{A}}^{\underline{E} \parallel \hat{\underline{n}}} + \overbrace{\int_{\substack{\text{base}\\2}} \underline{E} \cdot d\underline{A}}^{\underline{E} \parallel \hat{\underline{n}}} + \overbrace{\int_{\substack{\text{sup.}\\\text{laterale}}} \underline{E} \cdot d\underline{A}}^{\underline{E} \perp \hat{\underline{n}}}$$

$$= \int_{\substack{\text{base}\\1}} E\, dA + \int_{\substack{\text{base}\\2}} E\, dA + 0 = E \int_{\substack{\text{base}\\1}} dA + E \int_{\text{base } 2} dA = E \cdot A + E A = 2EA$$

A = area di base 1 o base 2

$$\oint_{\text{cilindro}} \underline{E} \cdot d\underline{S} = \frac{Q_{\substack{\text{carica}\\\text{nel piano}}}}{\varepsilon_0} = \frac{\sigma \cdot A}{\varepsilon_0}$$

Q^{int}

σ: carica/sup.

$$2E\cancel{A} = \frac{\sigma \cdot \cancel{A}}{\varepsilon_0} \; ; \quad E = \frac{\sigma}{2\varepsilon_0}$$

$$[\sigma] = C/m^2$$