In[∘]:=

# Utility Function

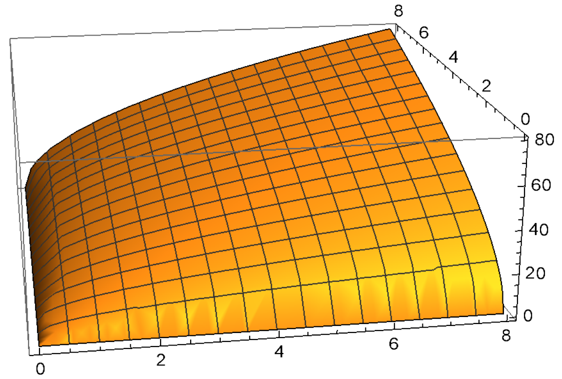
$U = 10\sqrt{X * Y}$

Utility function

$10\sqrt{X\,Y}$ con $X >> 0$ & $Y >> 0$ (ipotesi)

In[∘]:= Plot3D[U, {X, 0, 8}, {Y, 0, 8}]

Out[∘]=




In[∘]:= $H = \{\{\partial_{X,X}U, \partial_{X,Y}U\}, \{\partial_{Y,X}U, \partial_{Y,Y}U\}\}$

Out[∘]= $\left\{\left\{-\frac{5\,Y^2}{2\,(X\,Y)^{3/2}}, -\frac{5\,X\,Y}{2\,(X\,Y)^{3/2}} + \frac{5}{\sqrt{X\,Y}}\right\}, \left\{-\frac{5\,X\,Y}{2\,(X\,Y)^{3/2}} + \frac{5}{\sqrt{X\,Y}}, -\frac{5\,X^2}{2\,(X\,Y)^{3/2}}\right\}\right\}$

In[∘]:= MatrixForm[H]

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{5\,Y^2}{2\,(X\,Y)^{3/2}} & -\frac{5\,X\,Y}{2\,(X\,Y)^{3/2}} + \frac{5}{\sqrt{X\,Y}} \\ -\frac{5\,X\,Y}{2\,(X\,Y)^{3/2}} + \frac{5}{\sqrt{X\,Y}} & -\frac{5\,X^2}{2\,(X\,Y)^{3/2}} \end{pmatrix}$$

In[∘]:= Det[H]

Out[∘]= 0

Negative semi definite?

## Marshallian demands and optimal $\lambda$

In[∘]:= $L = U - \lambda\,(p_1 * X + p_2 * Y - R)$

Out[∘]= $10\sqrt{X\,Y} - \lambda\,(-R + X\,p_1 + Y\,p_2)$

In[∘]:= Solve[{$\partial_X L$ == 0, $\partial_Y L$ == 0, $\partial_\lambda L$ == 0}, {X, Y, $\lambda$}]

$$\left\{\left\{X \to \frac{R}{2\,p_1}, Y \to \frac{R}{2\,p_2}, \lambda \to \frac{5\sqrt{\frac{R^2}{p_1\,p_2}}}{R}\right\}\right\}$$

## Hicksian demands and optimal $\mu = \lambda^{-1}$ (verify)

$\Lambda = p_1 * X + p_2 * Y - \mu\left(10\sqrt{X * Y} - W\right)$ dove W è un livello dato di utilità

Out[∘]= $-\left(-W + 10\sqrt{X\,Y}\right)\mu + X\,p_1 + Y\,p_2$

In[ ]:= $\text{Solve}[\{\partial_X \Lambda == 0, \partial_Y \Lambda == 0, \partial_\mu \Lambda == 0\}, \{X, Y, \mu\}]$

Out[ ]= $$\left\{\left\{X \to -\frac{W\sqrt{p_2}}{10\sqrt{p_1}}, Y \to -\frac{W\sqrt{p_1}}{10\sqrt{p_2}}, \mu \to -\frac{1}{5}\sqrt{p_1}\sqrt{p_2}\right\}, \left\{X \to \frac{W\sqrt{p_2}}{10\sqrt{p_1}}, Y \to \frac{W\sqrt{p_1}}{10\sqrt{p_2}}, \mu \to \frac{1}{5}\sqrt{p_1}\sqrt{p_2}\right\}\right\}$$

**Prendiam solo soluzioni > 0.**

# Indirect Utility , Roy and Hotelling – Wold

In[ ]:= $$V = 10\sqrt{\frac{R}{2\,p_1} * \frac{R}{2\,p_2}}$$

$$5\sqrt{\frac{R^2}{p_1\,p_2}}$$

**Roy :**

In[ ]:= $$x = \frac{-\partial_{p_1} V}{\partial_R V}$$

Out[ ]= $$\frac{R}{2\,p_1}$$

**Hotelling – Wold :**

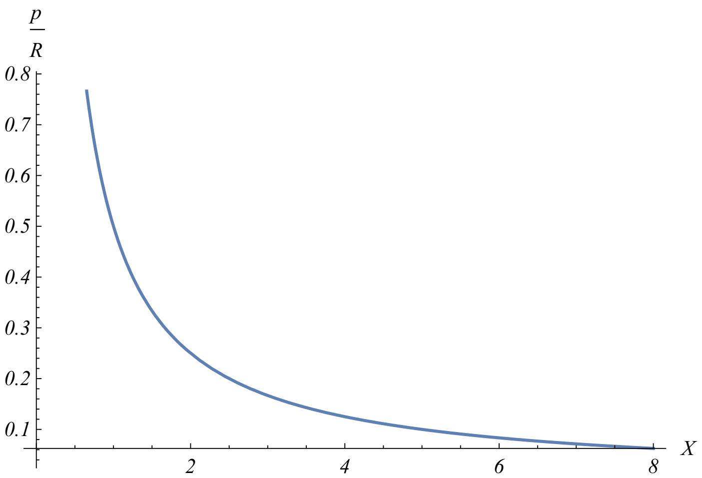
In[ ]:= $$\frac{p_1}{R} = \frac{\partial_X U}{X * \partial_X U + Y * \partial_Y U}$$

Out[ ]= $$\frac{1}{2\,X}$$

In[ ]:= $\text{Plot}\left[\frac{1}{2\,X}, \{X, 0, 8\}\right]$

In[ ]:= `Show[%1, AxesLabel → {HoldForm[X], HoldForm[p/R]}, PlotLabel → None, LabelStyle → {FontFamily → "Times New Roman", 11, GrayLevel[0], Italic}]`

Out[ ]=




# Expenditure function

$$M = p_1 \frac{W\sqrt{p_2}}{10\sqrt{p_1}} + p_2 \frac{W\sqrt{p_1}}{10\sqrt{p_2}}$$ **dove W è un livello dato di Utilità (vedi sopra)**

Out[∘]= $\frac{1}{5} W \sqrt{p_1}\ \sqrt{p_2}$

In[∘]:= **Shephard**

$g = \partial_{p_1} M$

Out[∘]= $\frac{W\sqrt{p_2}}{10\sqrt{p_1}}$

In[∘]:= $j = \partial_{p_2} M$

Out[∘]= $\frac{W\sqrt{p_1}}{10\sqrt{p_2}}$

**Slutsky**

In[∘]:= $\partial_{p_1} g$

Out[∘]= $-\frac{W\sqrt{p_2}}{20\, p_1^{3/2}}$

In[∘]:= $\partial_{p_2} g$

Out[∘]= $\frac{W}{20\sqrt{p_1}\ \sqrt{p_2}}$

In[∘]:= $\partial_{p_2} j$

Out[∘]= $-\frac{W\sqrt{p_1}}{20\, p_2^{3/2}}$

In[∘]:= $\partial_{p_1} j$

Out[∘]= $\frac{W}{20\sqrt{p_1}\ \sqrt{p_2}}$

In[∘]:= $S = \left\{\left\{-\frac{W\sqrt{p_2}}{20\, p_1^{3/2}}, \frac{W}{20\sqrt{p_1}\ \sqrt{p_2}}\right\}, \left\{\frac{W}{20\sqrt{p_1}\ \sqrt{p_2}}, -\frac{W\sqrt{p_1}}{20\, p_2^{3/2}}\right\}\right\}$

Out[∘]= $\left\{\left\{-\frac{W\sqrt{p_2}}{20\, p_1^{3/2}}, \frac{W}{20\sqrt{p_1}\ \sqrt{p_2}}\right\}, \left\{\frac{W}{20\sqrt{p_1}\ \sqrt{p_2}}, -\frac{W\sqrt{p_1}}{20\, p_2^{3/2}}\right\}\right\}$

In[∘]:= **Det[S]**

Out[∘]= 0

In[∘]:= **MatrixForm[S]**

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{W\sqrt{p_2}}{20\, p_1^{3/2}} & \frac{W}{20\sqrt{p_1}\ \sqrt{p_2}} \\ \frac{W}{20\sqrt{p_1}\ \sqrt{p_2}} & -\frac{W\sqrt{p_1}}{20\, p_2^{3/2}} \end{pmatrix}$$

In[•]:= **Eigenvalues[S]**

$\left\{0, \frac{W\left(-p_1^2 - p_2^2\right)}{20\, p_1^{3/2}\, p_2^{3/2}}\right\}$ **che è coerente con Det = 0**

**NegativeSemidefiniteMatrixQ[S] ?**
**Si, se tutti i p > 0 e W ≥ 0. A cosa corrispondono gli elementi diagonale principale ?**

## Expenditure function = inverse indirect utility

In[2]:= **Solve**$\left[\frac{1}{5} W \sqrt{p_1}\ \sqrt{p_2} == R, W\right]$

Out[2]= $\left\{\left\{W \to \frac{5 R}{\sqrt{p_1}\ \sqrt{p_2}}\right\}\right\}$