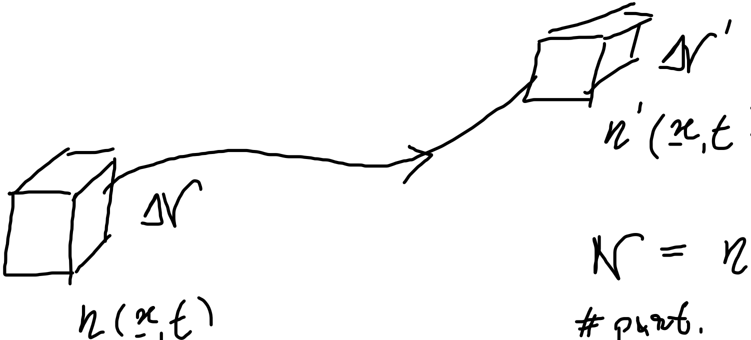
$n_\alpha(\underline{x},t)$ $\quad\quad$ $\underline{u}(\underline{x},t)$

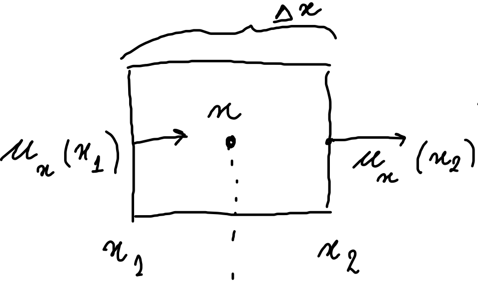


$N = n\Delta V = \text{const}$

\# part.

$$\frac{dN}{dt} = 0 \qquad \frac{dN}{dt} = \frac{dn}{dt}\Delta V + n\frac{d}{dt}\Delta V$$

$$\frac{d}{dt}(\Delta x \Delta y \Delta z) = \frac{d\Delta x}{dt}\Delta y \Delta z + \Delta x \frac{d}{dt}\Delta y \Delta z + \Delta x \Delta y \frac{d}{dt}\Delta z$$



$$x_1 = x - \frac{\Delta x}{2} \qquad x_2 = x + \frac{\Delta x}{2}$$

$$x_2' = x_2 + u_x(x_2)\Delta t$$

$$x_1' = x_1 + u_x(x_1)\Delta t$$

$$\Delta x' = x_2' - x_1' = \underset{\sim}{x_2} + u_x(x_2)\Delta t \underset{\sim}{- x_1} - u_x(x_1)\Delta t = \Delta x + \Delta t\left[u_x(x_2) - u_x(x_1)\right]$$

$$u_x(x_2) = u_x\left(x + \frac{\Delta x}{2}\right) \simeq u_x(x) + \frac{\partial u_x}{\partial x}\frac{\Delta x}{2}$$

$$u_x(x_1) = u_x\left(x - \frac{\Delta x}{2}\right) \approx u_x(x) - \frac{\partial u_x}{\partial x}\frac{\Delta x}{2}$$

$$u_x(x_2) - u_x(x_1) = \frac{\partial u_x}{\partial x}\Delta x$$

$$\Delta x' = \Delta x + \Delta t \frac{\partial u_x}{\partial x} \Delta x$$

$$\underbrace{\frac{\Delta x' - \Delta x}{\Delta t}}_{\frac{d}{dt}(\Delta x)} = \frac{\partial u_x}{\partial x} \Delta x$$

Per analogia

$$\frac{d}{dt}(\Delta y) = \frac{\partial u_y}{\partial y} \Delta y \qquad \frac{d}{dt}(\Delta z) = \frac{\partial u_z}{\partial z} \Delta z$$

$$\frac{d}{dt}(\Delta V) = \frac{\partial u_x}{\partial x} \underbrace{\Delta x \Delta y \Delta z} + \frac{\partial u_y}{\partial y} \underbrace{\Delta x \Delta y \Delta z} + \frac{\partial u_z}{\partial z} \underbrace{\Delta x \Delta y \Delta z}_{\Delta V}$$

$$= \Delta V \left[\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right] = \Delta V \cdot \underline{\nabla} \cdot \underline{u}$$

$$\frac{d}{dt} n(\underline{x},t) = \frac{\partial n}{\partial x} \cdot u_x + \frac{\partial n}{\partial y} u_y + \frac{\partial n}{\partial z} u_z + \frac{\partial n}{\partial t} =$$

$$= (\underline{u} \cdot \underline{\nabla}) n + \frac{\partial n}{\partial t}$$

$$\frac{dN}{dt} = 0$$

$$\Downarrow$$

$$\left[(\underline{u} \cdot \underline{\nabla}) n + \frac{\partial n}{\partial t} \right] \cancel{\Delta V} + n \cancel{\Delta V} \cdot \underline{\nabla} \cdot \underline{u} = 0$$

$$\boxed{\frac{\partial n}{\partial t} + \underline{\nabla} \cdot (n \underline{u}) = 0}$$

<u>Cons. qtà moto</u>

$$\frac{d}{dt}(\text{qtà di moto}) = \#\text{forze elemento fluido}$$

$$\text{q.tà di moto} = m\,\underline{u} \cdot \underbrace{(\#\,\text{particelle in } \Delta V)}_{N} = m\,\underline{u}\, n\, \Delta V$$

$$\frac{d}{dt}\left(m\,\underline{u}\, \underbrace{n\,\Delta V}\right) = m\, n\, \Delta V \frac{d\underline{u}}{dt}$$

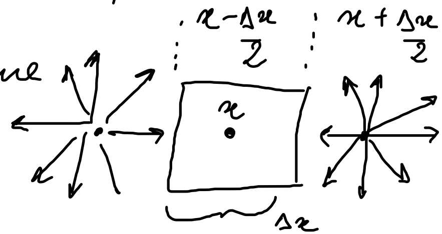
Forze

1) Forza Lorentz $\underline{F} = q\left(\underline{E} + \underline{u} \times \underline{B}\right)$ su una carica

$$\underline{F}_{tot} = N \cdot \underline{F} = n\,\Delta V\, q\left(\underline{E} + \underline{u} \times \underline{B}\right)$$

2) Forza dovuta alla pressione

p isotropa



$$F_x = F_{x-\frac{\Delta x}{2}} - F_{x+\frac{\Delta x}{2}}$$

$$F_x = F_{x-\frac{\Delta x}{2}} - F_{x+\frac{\Delta x}{2}} = p\left(x-\frac{\Delta x}{2}\right)\underbrace{\Delta y \Delta z}_{\text{Sup.}} - p\left(x+\frac{\Delta x}{2}\right)\Delta y \Delta z$$

$$\simeq \left[\cancel{p(x)} - \frac{\partial p}{\partial x}\frac{\Delta x}{2}\right]\Delta y \Delta z - \left[\cancel{p(x)} + \frac{\partial p}{\partial x}\frac{\Delta x}{2}\right]\Delta y \Delta z$$

$$= -\frac{\partial p}{\partial x}\Delta V$$

$$F_y = -\frac{\partial p}{\partial y}\Delta V \qquad F_z = -\frac{\partial p}{\partial z}\Delta V$$

$$\underline{F} = -(\underline{\nabla} p)\Delta V$$

3) Collisioni $e - i$

$$\underline{F}_{\substack{coll\\ e,i}} = -\left(\overline{\nu}\, m_e(\underline{u}_e - \underline{u}_i)\right)\underbrace{n\Delta V}_{N} \quad \alpha\ (\underline{u}_e - \underline{u}_i)$$

$$\underline{F}_{e,i} + \underline{F}_{i,e} = 0$$

$$\underline{F}_{i,e} = -\underline{F}_{e,i}$$

$$m_e n_e \frac{d\underline{u}_e}{dt} = -n_e e(\underline{E} + \underline{u}_e \times \underline{B}) - \underline{\nabla} p_e - m_e \nu_{ei} (\underline{u}_e - \underline{u}_i)$$

$$m_i n_i \frac{d\underline{u}_i}{dt} = Z n_i e(\underline{E} + \underline{u}_i \times \underline{B}) - \underline{\nabla} p_i + m_e \nu_{ei} (\underline{u}_e - \underline{u}_i)$$

$$\frac{\partial n_e}{\partial t} + \underline{\nabla} \cdot (n_e \underline{u}_e) = 0 \qquad \frac{\partial n_i}{\partial t} + \underline{\nabla} \cdot (n_i \underline{u}_i) = 0$$

$$\left.\begin{matrix} n_e & n_i \\ \underline{u}_e & \underline{u}_i \end{matrix}\right\} \quad \rho = -n_e e + Z e n_i \text{ (dens. carica)}$$

$$\underline{j} = \underline{j}_e + \underline{j}_i = -e n_e \underline{u}_e + Z e n_i \underline{u}_i$$

$$\underline{j}_\alpha = n_\alpha q_\alpha \underline{u}_\alpha$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \varepsilon_0 \qquad \underline{\nabla} \times \underline{E} = -\partial \underline{B} / \partial t$$

$$\underline{\nabla} \cdot \underline{B} = 0 \qquad \underline{\nabla} \times \underline{B} = \mu_0 \underline{j} + \varepsilon_0 \mu_0 \partial \underline{E} / \partial t$$

~~Aggiungo~~

$$\frac{\underline{\nabla} p}{p} = \gamma \frac{\underline{\nabla} n}{n}$$

Trasf. isoterma $T = \text{cost}$

$$pV = n_{mol} RT = N k_B T$$

$$p = n k_B T$$

Isoterma

$$\underline{\nabla} p = k_B T \underline{\nabla} n \; ; \quad \frac{\underline{\nabla} p}{p} = \frac{\cancel{k_B T} \underline{\nabla} n}{n \cancel{k_B T}} = \frac{\underline{\nabla} n}{n}$$

$$\gamma = 1$$

Trasf. adiabatica

$$\gamma = \frac{2 + f}{f}$$

$f \rightarrow$ # gradi di libertà

$$dU = \cancel{\delta Q} + \delta L \qquad \delta Q = 0 \text{ trasf. adiab.}$$

U en. interna gas/plasma

$$U = \overbrace{f \cdot \frac{k_B T}{2}}^{\langle E \rangle} N \qquad pV = N k_B T \; ; \; d(pV) = N k_B dT \quad \delta L = p\, dV$$

$$dU = -p\, dV$$

$$dU = \frac{f}{2} k_B N \, dT \underset{\text{I pr.}}{=} -p \, dV \qquad \text{eq. stato} \qquad dT = \frac{d(pV)}{k_B N}$$

$$\frac{f}{2} \cancel{k_B N} \frac{d(pV)}{\cancel{N k_B}} = -p \, dV$$

$$dp \, V + \underline{dV \cdot p} = -\frac{2}{f} p \, dV \; ; \qquad V \, dp = -\overbrace{\frac{2+f}{f}}^{\gamma} \cdot p \, dV \; ; \qquad \frac{dp}{p} = -\gamma \frac{dV}{V}$$

$$N = nV \; ; \quad dN = 0 = d(nV) \; ; \quad dn \cdot V + n \, dV = 0 \; ;$$

$$-\frac{dn}{n} = \frac{dV}{V} \qquad \frac{dp}{p} = \gamma \frac{dn}{n}$$

$\underline{B} = \overrightarrow{\text{const}}$ $\quad \underline{E} \neq \underline{0}$ $\quad$ No collisioni $\quad \underline{F} = q\underline{E}$

Singola partic. $\quad \underline{v}_D = \dfrac{\underline{F} \times \underline{B}}{qB^2} = \dfrac{\underline{E} \times \underline{B}}{B^2}$

$\underline{u} = \overrightarrow{\text{const}}$: $\dfrac{d\vec{u}}{dt} = 0$

eq. cons. qtà di moto $\quad 0 \approx \left[nq \left(\underline{E} + \underline{u} \times \underline{B} \right) - \underline{\nabla} p \right] \times \underline{B}$

$$0 \approx nq \left(\underline{E} \times \underline{B} \right) + nq \left(\underline{u} \times \underline{B} \right) \times \underline{B} - \underline{\nabla} p \times \underline{B}$$

$$\left(\underline{C} \times \underline{B} \right) \times \underline{A} = \left(\underline{A} \cdot \underline{C} \right) \underline{B} - \left(\underline{A} \cdot \underline{B} \right) \underline{C}$$

$$\underline{u} \quad \underline{B} \quad \underline{B} \quad \cancel{\underline{u} \cdot \underline{B}} - B^2 \underline{u}$$

$$\underline{u}_\perp \quad \underline{B}$$

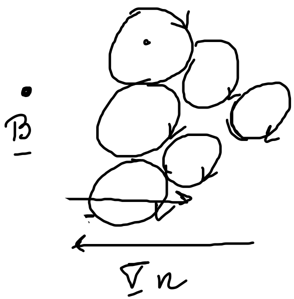
$$0 \approx nq \left(\underline{E} \times \underline{B} \right) - nqB^2 \underline{u} - \underline{\nabla} p \times \underline{B}$$

$$\underline{u}_\perp = \cancel{\frac{\underline{E} \times \underline{B}}{B^2}} - \frac{\underline{\nabla} p \times \underline{B}}{nqB^2}$$

$$\underline{u}_{\text{diamagnetica}} = -\frac{\underline{\nabla} p \times \underline{B}}{n q B^2}$$

Se $T = \text{const}$ $\quad p = nT \quad : \quad \underline{\nabla} p = T\, \underline{\nabla} n$

Ioni



$\underline{u}_{\text{deriva}} \propto -(\underline{\nabla} n \times \underline{B})$

Equazioni a 1 fluido (o magnetoidrodinamica)

1) T $\quad v_{th_i} \ll v_{th_e} \quad v_{th} = \sqrt{\frac{2T}{m}} \quad \frac{m_e}{m_i} \simeq \frac{1}{2000} \quad v_{th_e} \simeq 40\, v_{th_i}$

$\tau_e \sim a / v_{th_e} \quad \tau_i \sim \frac{a}{v_{th_i}} \quad \tau_e \ll \tau_i \quad \tau \sim \tau_i$

$m_e \simeq 0$

2) $T \sim keV$ $\quad v_{th_i} \sim 10^6\ m/s$

$$v_{th_i} \ll c$$

$$\underline{\nabla} \times \underline{B} = \cancel{\mu_0 \underline{j}} + \cancel{\varepsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}}$$

$$\frac{E}{B} \sim c$$

$$\frac{\left\| \varepsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t} \right\|}{\left\| \underline{\nabla} \times \underline{B} \right\|} \sim \frac{\frac{1}{c^2} \, {}^{E}/_{\tau_i}}{{}^{B}/_{L}} \approx \frac{1}{\cancel{c^2}} \cancel{c} \cdot \frac{L}{\tau_i} \approx \frac{v_{th_i}}{c} \ll 1$$

3) $-e n_e + Z e n_i \approx 0 \qquad n_e \approx Z n_i \qquad Z = 1$ (pl. di H)

$$n_e \approx n_i$$

$$\frac{\partial n_e}{\partial t} + \underline{\nabla} \cdot (n_e \underline{u}_e) = 0$$

$$\frac{\partial}{\partial t} n_i + \underline{\nabla} \cdot (n_i \underline{u}_i) = 0$$

$$0 \approx -e n_e (\underline{E} + \underline{u}_e \times \underline{B}) - \underline{\nabla} p_e - m_e n_e \nu_{ei} (\underline{u}_e - \underline{u}_i)$$

$$m_i n_i \frac{d\underline{u}_i}{dt} = Z e n_i (\underline{E} + \underline{u}_i \times \underline{B}) - \underline{\nabla} p_i + m_e n_e \nu_{ei} (\underline{u}_e - \underline{u}_i)$$

$$\underline{\nabla} \times \underline{E} \quad -\partial \underline{B} / \partial t \qquad \underline{\nabla} \times \underline{B} = \underbrace{e \mu_0 n_e (\underline{u}_i - \underline{u}_e)}$$

$$\underline{\nabla} \cdot \underline{B} = \underline{0} \qquad Z n_i \approx n_e \qquad \mu_0 \left[-e n_e \underline{u}_e + Z e \underline{u}_i n_i = e n_e (\underline{u}_i - \underline{u}_e) \right]$$

Variabili di singolo fluido:

$$\rho_m = m_e n_e + m_i n_i \approx m_i n_i \qquad \underline{u} = \frac{\underline{E} \times \underline{B}}{B^2} - \frac{\underline{\nabla} p \times \underline{B}}{q n B^2}$$